Original Research Article

An Application Using Stochastic Approximation Method for Improvement Specific Loss System

Abstract- In this paper, an application using the modified stochastic approximation procedure which studied to answer the question for Robbins-Monroe procedure. The modified procedure depends on a new form. We use the case of loss system obey Negative Binomial distribution. The efficiency of the proposed procedure is calculated to determine the ways to improve the mentioned loss system, the results which are obtained show that our procedure can serve as a model of stochastic approximation with delayed observations. This new topic can be applied in many fields such as the biological, medical, life time experiments, and some industrial projects, to increase the production, where items are realized after random time delays.

Keywords: *Delayed Observations; Negative Binomial Distribution; Robbins-Monro procedure; Stochastic Approximation.*

1. Introduction

It is known that stochastic approximation is considered a widespread procedure for both the root-seeking problem for unknown functions and the solution of a system of equations when the value of the respective function can only be measured with experimental errors at recursively determined points [15].

For solving the present problem, Robbins and Monroe proposed a recursive algorithm to approximate the sought for root .This algorithm follows the pioneer work of stochastic approximation. There are a large amount of applications to practical problems and researches that work on theoretical issues.

A modeling and analysis of stochastic approximation procedure have become an important field for performance analysis [4, 5, 6, 7, 8, 16, 17].The Robbins and Monroe stochastic approximation were applied in clinical applications to find the optimal dose [2]. Also, by using the idea of averaging [14] that proposed a new stochastic approximation algorithm establishes the almost surly for this procedure by

[1]. In another direction [13], is applied stochastic approximation method on confidence interval.

The proposed question for Robbins and Monroe is, whether stochastic approximation can be applied when the results of experiments become known only after a random time delay, as many biological or life time experiments.

Dupač and Herkenrath[3] , proposed to answer this question by applying Robbins-Monroe stochastic approximation procedure in the presence of delayed observations and represented the simplest model for discrete waiting time using geometric distribution. In [8] ,Mahmoud and Rasha have also answered this question using a new application, the Robbins and Monroe stochastic approximation is applied in the presence of compound delayed observation. Also, in [9], the estimating of random time delay distribution of the compound observations was found by applying Geometric time delay distribution by observations of the stochastic approximation procedure. Supplement to this direction, [10], assuming that the random time delay distribution of observations, with delayed components, is decreasing and independent of the delay distribution of their components. In [11], stochastic approximation procedure is modified to be applicable in the presence of delayed groups of delayed multiservice observations. This procedure is depending on a new base which concerning the relation between service time of the group and service time of its components

The organization in our paper as follow, first we describe our proposed procedure. In section 2, we compute the efficiency of the modified stochastic approximation procedure in section 3. Then in section 4 we present the methodology of our procedure. Finally the results and discussion are included in section 5.

2. Description of the proposed procedure

Let $f(x)$ be a real function of real variable, $x_o \in R$ apply the following condition:

$$f(x) < f(x_0) = 0 < f(x_m)$$

$\forall\, x < x_o$, $x_m > x_o$, x_o is the zero point of the function $f(x)$ is to be found .

Assume that $f(x)$ is observable, where there is an experimental error that found at any point of $x \in R$. The experiment can be performed for each time unit n=1,2,3……, Robbins –Monroe stochastic approximation procedure is used to obtain x_o ,that is by following these steps :

Allocate K parallel series according to a rule to be described later, x_z^k is the point at Z^th experiment of k^{th} series; let y_z^k denote the result of that experiment then:

$$y_z^k = f(x_z^k) + e_z^k(x_z^k)\ .$$

Where, $\forall\, k = 1,2, \dots, K,\ Z = 1,2, \dots..$ $\ni e_z^k(x),\ x \in R$ are independent centered random functions.

After each time unit we observe each series, assume that $\forall\, k\,, z$ there is delays $\mathrm{d}_{\mathrm{z}}^{\mathrm{k}}$, at each observation $\mathrm{y}_{\mathrm{z}}^{k}$ are considered independent and identically distributed integer valued random , also independent of $\mathrm{e}_{\mathrm{z}}^{k}$'s .

Let x_1^k , (k=1,2,…… ,K) is the starting point of an experiment . Here, we assume that in each series the random function $\mathrm{e}_{\mathrm{z}}^{k}$(x) and the function $f(x)$ together are considered as fulfill conditions to guaranty the convergence of x_{Z}^k to x_0 as $Z \to \infty$ almost surly to asymptotic normality with parameter $(0\ ,\ \sigma)$ of normal approximations $\sqrt{\mathrm{Z}\left(x_{\mathrm{Z}}^{(k)} - x_0\right)}$ see [12] for such set of conditions.

Let K- series of the proposed loss system be either' ready 'or 'waiting ' ,at the point x_{Z}^k ($k = 1,2, \dots, K$) immediately before time t . At the beginning, i . e before time $t = 1$, all server are ready and all t_k equal to one. At time instant t, it is made an experimental point $x_{t_i}^i$ where i is one of the ready series e . g , a randomly chosen one. If the observation $y_{t_i}^i$ is immediately realized (i .e the result of this series achieve that condition) :

$$c \geq s \text{, where, } c \text{ is the number of realized points from the } n \text{ units which entering the system, } c, s \leq n \tag{1}$$

Then the series i is ready again at point

$$x_{t_{i+1}}^i = x_{t_i}^i - a_{t_i}\, y_{t_i}^i$$

after a unite time , where $\{\, a_i\}$ is a fixed sequence defining Robbins – Monroe procedure , a_j =a/j is considered one form of the a_i choice .

If the observation $y_{t_i}^i$ is delayed (i . e the result from the i^{th} series did not achieve the condition (1) then the series i will be waiting at the point $x_{t_i}^i$. If the observations $y_{t_j}^j$ are also realized [achieve the condition (1)] at time instant t in some series that were waiting before , then the series become ready after time t , at respective points

$$x_{t_{j+1}}^j = x_{t_j}^j - a_{t_j}\, y_{t_j}^j$$

The ready states, waiting and pointing of all the other series remain unaltered after time t, that is equal to those before time t . If there is no series ready before time t, no experiment is made at this time t. A loss of the efficiency of the procedure being incurred . At the same time the other rules for time t, were described before.

According to the previous description of the proposed system is considered a loss system of Negative Binomial service time distribution, which presents a model for discrete waiting times , i.e assume for all k and the random time delay D

$$p(D = d) = \binom{d+c-1}{c-1} p^c \ q^d , d = 0,1, \dots \dots . . , c = s, s+1, \dots . , n$$

Where c is the number of parts which realized without delay.

It is plained that when c=1 then the service system is equivalent to the service system that was given by Dupač and Herkenrath [3] ,where the geometric distribution was applied. In our service system the stats, $[0, 1, 2 , \dots \dots \dots . , K]$, are just the number of servers that are free (i . e achieve the condition) , representing a Markov chain .The transition probabilities are

$$P_{jk} = \begin{cases} \binom{K}{k} p_N^k \ q_N^{K-k} & , j = 0,1 \ ; k = 0,1,2, \dots . K \\ 0 & , j = 2, \dots , K \ ; k = 0, \dots , j-2 \\ \binom{K-j+1}{k-j+1} p_N^{k-j+1} \ q_N^{K-k} & j = 2, \dots \dots , K \ ; k = j-1, \dots . , K \end{cases} \quad (2)$$

$$, \ p_N = \sum_{i=0}^{s-1} \binom{n}{i} \ p^i \ q^{n-i}$$

$$q_N = \sum_{i=s}^{n} \binom{n}{i} \ p^i \ q^{n-i}$$

it explains that $p_N + q_N = 1$.

3. Computation of the Efficiency of the proposed Stochastic Approximation Procedure

The states {0,1,..,k}, of the described system, are just the number of servers free (at time $n - 0$).

For the resulting matrix from (2), we can observe the following conditions:

i) All states are irreducible closed sets; therefore they contain persistent non-null states.

ii) All states have period 1 because,

$$p_{jj}(1) > 0, \quad \forall j.$$

From the previous conditions, all states are ergodic [4], then there is a unique stationary distribution π that can be calculated by solving the system of equations

$$P^{\mathrm{T}} \pi = \pi \quad (3)$$

together with the added requirement, $1^{T}\pi = 1$,where T denotes the transpose of the matrix , π is the stationary distribution matrix of the Markov chain , P is the matrix of the transition probabilities.

4. Methodology

To solve system (3) we use the following steps:

1. assume the values n, s, k to repair the transition matrix
2. Form system (3) we use the transition matrix p.
3. One of the remaining equation can be deleted, another one is to be added, namely the requirement $\sum_{\forall\alpha} \pi_\alpha$ =1.
4. The resulting system is being solved to obtain the values of $\pi_\alpha \ \forall\alpha = 0,1, \dots, K$.
5. The solution of π_α is used to compute the efficiency e of the proposed procedure

$$e = 1 - \pi_\alpha.$$

5. Result and Discussion

Table1. gives the asymptotic efficiency for the proposed procedure, by assumed that $p = 0.1, 0.2, \dots\dots, 0.9$, that helps for assuming $p_N, E(d)$; in case $k = 1,2, \dots 8$, $n = 8$, and the realized points $s = 3$. The adapter for choice of K, seems to be a good option. Also, we can note that the obtained efficiencies in the proposed procedure(e) is better than that in the Geometrical case (e_g) .

In Table2, it is shown that, for case $n = 8, s = 1$, for $k = 1,2, \dots 8$, the asymptotic efficiencies which obtained by applying the proposed procedure using Geometric distribution.

So, we can consider the Geometrical case is a special case of our procedure.

Finally, we expect that our proposal procedure can be helped for studying many experiments in different fields, especially, in scientific experiments.

Table1.Comparing the percentage asymptotic efficiencies of proposed procedure e with the percentage asymptotic efficiencies of Geometric distribution procedure e_g, with Parameter p; case $n = 8, s = 3$.

			$k = 2$		$k = 3$		$k = 4$		$k = 7$		$k = 8$	
p	p_N	$E(d)$	e	e_g	e	e_g	e	e_g	e	e_g	e	e_g
0.1	0.038	75.75	7.6	2.61	11.4	3.91	15.2	5.21	26.52	9.11	30.27	10.42
0.2	0.203	11.77	39.6	15.61	57.39	23.32	72.83	30.95	97.5	53.13	99.34	60.19
0.3	0.448	3.69	77.68	41.45	94.5	59.79	99.3	75.4		98.22		99.51
0.4	0.685	1.38	96	74.18	99.85	92.49		98.78				
0.5	0.856	0.51	99.66	95.1		99.77						
0.6	0.950	0.16	99.99	99.72								
0.7	0-989	0.03										
0.8	0.999	0.004										
0.9	1	0										

100 in all empty cells

Table2. Comparing the percentage asymptotic efficiencies of proposed procedure e with the percentage asymptotic efficiencies of Geometric distribution procedure e_g, with Parameter p; case $n = 8, s = 1$.

			$k = 2$		$k = 3$		$k = 4$		$k = 5$		$k = 6$		$k = 7$		$k = 8$	
p	p_N	$E(d)$	e	e_g	e	e_g	e	e_g	e	e_g	e	e_g	e	e_g	e	e_g
0.1	0.57	0.756	89.4	89.4	98.9	98.9	99.95	99.95								
0.2	0.83	0.202	98.5	98.5												
0.3	0.94	0.06														
0.4	0.98	0.017														
0.5	0.996	0.004														
0.6	0.999	0.007														
0.7	1	0														
0.8	1	0														
0.9	1	0														

100 in all empty cells

6.References

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