

# Original Research Article

## **Preservice Teachers' Content Knowledge and Learning Obstacles in Shape and Space**

### **Abstract**

*The notion of shape and space is the building block for geometry reasoning. Accordingly, it is expected that those trained to teach would possess substantial content knowledge for teaching planar and spatial notions. This study investigated the level of content knowledge and sources of learning obstacles encountered by preservice teachers prior to their first practice teaching of shape and space. The study adopted a cross-sectional survey design involving 757 second-year preservice teachers from 12 colleges of education in Ghana. Data were collected through written responses to basic shape and space tasks. Quantitative data were analyzed descriptively while qualitative data were analyzed thematically into matrix index. Participants' content knowledge were categorized into declarative, conditional and procedural knowledge types. The result shows that participants were largely operating at moderate levels of declarative, procedural and conditional content knowledge. However, participants demonstrated higher procedural knowledge than declarative and conditional knowledge. Tasks on prisms and pyramids were more difficult for preservice teachers than those on angles, triangles and quadrilaterals. The learning obstacles encountered were mostly didactical followed by epistemological with few being ontogenetic. The study recommends that tutors should employ investigative didactic strategies to promote the three content knowledge types identified and to address the epistemological and ontogenetic obstacles in preservice teachers' learning of shape and space.*

**Keywords:** *Declarative knowledge; Conditional knowledge; Procedural knowledge; Shape and space; Learning obstacles*

## **Introduction**

The notion of *Shape and Space* is central in mathematics education. In most basic mathematics curricular across the world, the content of shape and space forms the building block for developing students' mathematical reasoning, geometry thinking and visuo-spatial configuration for solving real world problems. In Ghana, shape and space is studied at the basic schools to enable students organize and use spatial relationships in solving real world problems (Akayuure, Asiedu-Addo & Alebna, 2016). In the basic school mathematics curriculum, students are expected to study definitions, properties, generalizations (declarative knowledge) and procedures (procedural knowledge), and their connections (conditional knowledge) in points and lines, angles and triangles, quadrilaterals, prisms and pyramids (Ministry of Education, 2012; Ministry of Education, Youth and Sports 2007).

Mathematics teachers are mandated with the professional responsibility of ensuring that learning takes place and the curriculum expectations for learning shape and space are met. To ensure this, the teacher must possess substantial content and pedagogical skills on shape and space to effectively facilitate the students' learning process. Unfortunately, literature suggests that the effectiveness of teacher education and training of preservice teachers remain an unresolved problem and far-fetched. In Ghana for example, it is argued that Geometry course at Colleges of Education does not develop the kind of content knowledge and exposure needed for preservice teachers to teach shape and space effectively at the basic school level (Armah, Cofie

& Okpoti, 2017). The argument makes reference to Chief Examiners' reports where Ghanaian preservice teachers appear to lack didactic conceptual structures of basic shapes and spatial relations (Institute of Education, 2017; 2014; 1995). For example, the Chief Examiners' report for 2015 Geometry paper revealed that, most preservice teachers were unable to discriminate between pyramids and prisms or rotational symmetries of shapes or determine the hierarchical classification of quadrilaterals (Ministry of Education, Ghana, 2015). These existing assessment reports are however inadequate for understanding the content knowledge of these preservice teachers. Therefore, there is need for an empirical research to understand preservice teachers' content knowledge and the learning obstacles in shape and space.

### **Statement of the Problem**

The essence of teacher education is to provide prospective teachers with the needed knowledge base for teaching. However, globally, there are concerns that majority of the preservice teachers turned out of the teacher education with limited requisite knowledge for teaching (Noto, Priatna, & Dahlan, 2019). These concerns have also been raised in the teaching of geometry course at the College of Education in Ghana (Armah, Cofie, & Okpoti, 2017). There are arguments that content knowledge acquired by preservice teachers for teaching basic concepts such as shape and space is quite inadequate. The inadequate knowledge for the teaching is attributed to some learning obstacles at the colleges of education in Ghana. While these arguments appear plausible, little research has been conducted to clearly understand the depth of content knowledge and sources of learning obstacles associated with shape and space among preservice teachers at the Colleges of Education in Ghana.

### **Purpose of the study**

The purpose of the study was to examine preservice teachers' content knowledge for teaching and learning obstacles associated with shape and space. Driven by this, the following research questions were formulated to guide the study:

1. What is the level of preservice teachers' content knowledge on basic shape and space?
2. What learning obstacles are identifiable in preservice teachers' written responses to content knowledge tasks on basic shape and space?

### **Significance of the study**

Since 2012, Ghana has been engaged in transforming teacher education and learning with the review of existing curriculum and upgrade of Colleges of Education from diploma to degree awarding colleges. The transformation drives the curriculum, development of new courses and adoption of creative teaching approaches to ensure the production of quality teachers to serve basic schools in Ghana. The study assess the impact of the reviewed pedagogies on preservice teachers' knowledge for teaching as well as the barriers of learning shape and space. It is hoped that the study would provide important information regarding the preservice teachers' content knowledge level and learning obstacles. This would direct policy practices and assist tutors to select and utilize suitable pedagogies to address learning obstacles and ensure quality learning by preservice teachers.

### **Theoretical background**

Preservice teachers' content knowledge has been framed and studied in different forms (Gogoe, 2009; Ball, Thames, & Phelps, 2008). Among these, Shulman was the first to categorize teacher knowledge into two dimensions; (1) content knowledge and (2) pedagogical knowledge. Several related mathematical knowledge frameworks have subsequently been proposed and studied. For example Ball, Thames and Phelps (2008) extended Shulman's pedagogical knowledge and proposed the mathematical knowledge for teaching. The frameworks however remain tentative because of some shortcomings related to teacher preparation and development processes. For example, content knowledge has not been studied as a multi-dimensional construct to reveal aspects that need improvement or intervention. In this study, the developmental precedence of knowledge types is applied to examine preservice teachers' content knowledge of shape and space. Alexander and Judy (1988) categorize such content knowledge into declarative (knowing that), procedural knowledge (knowing how to) and conditional (knowing why). According to modern cognitive sciences, understanding the developmental precedence of different facets of knowledge is key to identifying instructional interpolations and learning obstacles (Cevirgen, 2012; Schneider, Rittle-Johnson & Star, 2011; Alexander & Judy, 1988).

Cognitive psychologists view declarative knowledge as store of facts and experiences in the long-term memory (Schneider, Rittle-Johnson & Star, 2011) and further branded it as "knowing-that" or "knowing what" (Yilmaz & Yalcin, 2012). Defined differently, Schneider, Rittle-Johnson and Star (2011) viewed declarative knowledge as an explicit knowledge of facts, of generalization and of theories or

hypotheses filtered by generalized opinions, beliefs and attitudes. To illustrate, if a learner can label and name various forms of prisms, list or identify properties of prisms and distinctively describe prisms using appropriate terminologies, then such a learner demonstrates declarative knowledge (Figure 1). Declarative knowledge is often transferred into the long-term memory for quick recall upon demand.

According to Yilmaz and Yalcin (2012), procedural knowledge is described as the set of knowing how to do or take step-by-step instructions to accomplish certain tasks. It relates to doing or solving a problem by assembling knowledge of concepts, facts, theories; making rational predictions of rules and procedures; applying critical judgments of appropriate procedures to follow; arriving at conclusions for choice procedures; deciding on the best course of actions, and executing the actions successfully (Aydin & Ubuz, 2010). This study views procedural knowledge as the knowledge of strategies, illustrations, sketches, algorithms and computational approaches needed to arrive at solutions to geometric problems (Figure 1).

Conditional knowledge is the interplay between conceptual knowledge and procedural knowledge and usually branded as *knowing why, when and where* to access facts and employ a procedure (Alexander & Judy, 1988). An individual who possesses conditional knowledge can determine the contexts and conditions under which other knowledge types are applicable in accomplishing tasks. Conditional knowledge essentially entails relational rules which are networks of if-then statements or condition-to-action sequences (Yilmaz & Yalcin, 2012; Aydin & Ubuz, 2010). For example, to answer the conditional question “*when does a*

*rhombus become a square?*” the learner should be able to state similar and dissimilar properties of both rhombus and square as well as the procedures needed to configure a rhombus into a square and vice versa. . For conditional knowledge, the preservice teachers should be able to justify, predict and explain why certain strategies or representations work in relation to basic shape and space (Aydin & Ubuz, 2010). It is operationalized in this study that conditional knowledge is based on the integration of relevant declarative and procedural knowledge as shown in Figure 1.

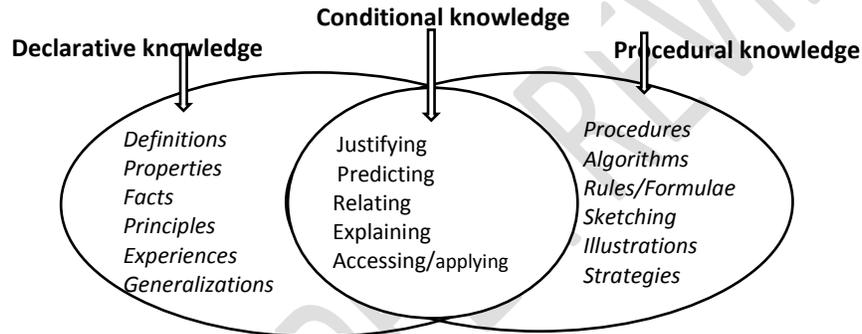


Figure 1 Content knowledge on shape and space (Author’s own construct)

Most existing studies in geometry treat geometry content knowledge as a uni-dimensional construct without regards to its inherent configurations (Cevirgen, 2012; Gogoe, 2009). Configuring geometry content knowledge into separate components could however elucidate hidden learning obstacles and direct instructional trajectories. In this study, content knowledge of shape and space is compartmentalized into declarative, conditional and procedural forms in order to investigate preservice teachers’ understanding levels and learning obstacles.

## **Learning obstacles in Geometry**

A learning obstacle is any idea, principle or procedure that one holds which works correctly in certain settings but becomes incidence of errors in other learning situations (Fuadiah, 2015). Their occurrences indicate that the student is studying or attempting to relate an existing concept to new concepts. Studies have shown that students often encounter these dilemma or obstacles when learning geometry (Riastuti, Mardiyana & Pramudya, 2017). Star & Stylianides (2013) and Haapalaso & Kadijevich (2000) cited learners' conceptual and procedural gaps in geometry, while Riastuti, Mardiyana and Pramudya (2017) identified poor spatial sense and inaccurate definitions as some problems in learning geometry. There are also specific obstacles linked to ineffective teaching styles and cognitive deficiencies in learning geometry (Noto, Priatna & Dahlan, 2019; Cevirgen, 2012).

However, within the broader theory of didactical situations, Brousseau (2002) categorized students' learning obstacles into ontogenetic, epistemological and didactical obstacles. The obstacles which relate to mental readiness or age-related cognition or limitation in earlier experience with learning material are called ontogenetic obstacles. These obstacles are built around psychological, cognitive and social readiness experienced by learners. For example, negative previous learning experience may affect the learner's readiness to be attentive in mathematics lesson. The obstacles that emanate from the structure of the complexity of mathematical content itself, its historical development and application are called epistemological obstacles (Fuadiah, 2015). They are construed as incomplete or faulty ways of thinking which are developmentally necessary or productive. Such obstacles which

often arise from context drawbacks can be difficult to avoid (Brousseau, 2002) and may stagnate an individual's knowledge acquisition (Noto, Priatna & Dahlan, 2019; Moru, 2009). The last category, the didactical obstacles, arise from learning which relate to teaching practices (Brousseau, 2002). They include the teacher's curriculum enactment, teaching processes, discourse attitudes, concepts illustrations, assessment practices and solution processes. While some learning obstacles have been identified in piece meal in different studies on shape and space concepts, this study contributes to the scarce research on the existence of broad learning obstacles in preservice teachers' learning of shape and space at college level.

### **Research methods**

#### *Design and sample*

The study adopted a cross-sectional survey design to examine preservice teachers' performance and learning obstacles when responding to knowledge base tasks on shape and space. The population comprised 3,861 and 8,697 second-year preservice teachers from 15 science/mathematics and 24 general programmes colleges of education in Ghana respectively (Institute of Education, 2017). These preservice teachers were exposed to the full content of geometry for teaching shape and space at basic school level and were preparing to undertake their teaching practicum. Five (5) and seven (7) colleges of education were drawn at random to represent the science/mathematics and the general programmes categories respectively. The selected colleges represent approximately 31% of 39 public colleges of education in Ghana. This proportion satisfied the recommended minimum random sample of 20%

needed for cross-sectional designs (Cohen, Manion & Morrison, 2007). All second-year preservice teachers of the selected colleges were considered for the study.

### *Instrument*

The study employed test as data collection instrument. In line with the shape and space contents ascribed in the Ghanaian basic mathematics curriculum and Colleges of Education (Institute of Education, 2014; Ministry of Education, 2012), the test comprised of tasks on angles, triangles, quadrilaterals, prisms and pyramids designed to measure preservice teachers' content knowledge. The tasks were designed to measure declarative, conditional and the procedural knowledge on shape and space. The declarative knowledge tasks focused on facts, names, and lists and involved the "what . . .", "how many . . ." and "name. . ." types of tasks. Procedural knowledge tasks involved "how to" problems such as the sketch of figures and use of rules and formulae in solving problems. Conditional knowledge tasks focused on understanding a network of condition-action sequences and predicting what happens "if" one of the variables in the sequence changes within the context of "if . . . then . . ." and "why . . ." relationship statements.

### *Validity and reliability*

To ensure representation and content validity of the test, an item specification table was created to align declarative, conditional and procedural tasks. The test items were thoroughly examined and scrutinized by two tutors at one of the colleges of Education in Ghana. The items were found to be appropriately aligned with the shape and space contents and were suitable for assess content knowledge. The Cronbach Alpha reliability coefficients were calculated for the items using the pilot

sample of 532. The analysis was performed separately for the items for three constructs and for the overall items. For the 5 items on declarative knowledge, the computed reliability coefficient was .837. For the 5 items of the conditional knowledge, the computed reliability coefficient was .919. For the 8 items of the procedural knowledge, the computed reliability coefficient was .913. The computation of Type C interclasses correlation coefficients using consistency definition also yielded average measure values of .794 and .954 at 95% confidence interval. The overall reliability coefficient of .897 was obtained for all 18 items. All the reliability values were greater than the recommended cut off value of .70 (Kline, 2015) and hence signified that the items were largely consistent and reliable.

*Data collection*

To collect data, permission was sought in writing to the principals of the selected Colleges of Education. When permission and access were granted, the researcher visited each colleges. In each college, verbal consent were sought from participating preservice teachers and those who agreed to take part in the study converged in their respective classrooms to answer the test items. The tutors voluntarily assisted the researcher to organize and distribute the test to participants. Most participants completed the test in 45 minutes. A total of 757 male and female preservice teachers’ scripts from two categories of academic programme of study were received for the study as shown in Table 1.

Table 1 Frequency distribution of participants’ gender and programme of study

Gender	Programme of Study	Total
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	General	Science	
Male	308	187	495
Female	95	167	262
Total	403	354	757

As shown in Table 1, there were relatively more males than females in the study. The male participants in the general programmes category were considerably more than their female participants. The distributions of participants by gender and by programme of study mirrored the population distribution at the college of education (Institute of Education, 2017). The sample and its distribution were therefore deemed representative of second-year preservice teachers at the Colleges of Education in Ghana.

#### *Data analysis*

The test scripts were scored using a scoring rubrics designed by the researcher. The scores were keyed into SPSS for further processing. Descriptive statistics were computed and displayed in tables. Also, to determine participants' knowledge levels, the scores were categorized into low, moderate and high levels and presented in percentages. Out of the maximum scores, participants who obtained scores from 0-3 (i.e. the lower 33.3% of the distribution of performance) were categorized as low. Those who obtained scores from 4-7 (i.e. occupying middle 33.4% of the distribution) were categorized as moderate while those who scored from 8-11 (upper 33.3% of the distribution) were categorized as high. These were used to answer research question one. For research question two, participants' written responses on

the test scripts were cataloged and characterized into ontogenetic, epistemological and didactical obstacles. Descriptors of each learning obstacles were put into a matrix of the three obstacles identified and interpreted.

## Results and discussion

### *1. Preservice teachers' content knowledge of basic shape and space*

The first research question sought to examine preservice teachers' basic content knowledge in shape and space. The analysis of data resulted into three knowledge types comprising declarative, procedural and conditional content knowledge of basic shape and space. Summary of descriptive statistics of scores obtained by participants are displayed in Table 2.

Table 2 Descriptive statistics of scores in content knowledge tasks on shape and space

<b>Knowledge Types</b>	<b>N</b>	<b>Mean(%)</b>	<b>St. Dev(%)</b>	<b>Possible Max(%)</b>
Declarative Knowledge	757	7.0(63.5)	2.10(19.1)	11(100)
Conditional Knowledge	757	4.8(44.0)	2.10(19.1)	11(100)
Procedural Knowledge	757	9.5(50.1)	4.07(21.4)	19(100)
<b>Overall</b>	<b>757</b>	<b>21.4(52.1)</b>	<b>6.65(16.2)</b>	<b>41(100)</b>

From Table 2, the mean score on declarative knowledge tasks was 7.0 with standard deviation of 2.10 reflecting 19.1% of the possible maximum score of 11. For conditional knowledge tasks, the mean score was 4.8 with standard deviation of 2.10 which corresponds to 19.1% of the possible maximum score of 11. Finally, for procedural knowledge tasks, the mean score was 9.5 with standard deviation of 4.07 which corresponds to 21.4% of the possible maximum score of 19. Participants' overall mean score of 21.4 with standard deviation of 6.65 reflects 52.1% of the possible maximum score of 41. The overall result shows that participants did averagely well in tasks on content knowledge of shape and space.

Further analysis examined the percentage distributions of the levels of preservice teachers' declarative, conditional and procedural knowledge in five main contents of shape and space. The results are categorized into low, moderate and high as shown in Table 3 to Table 5.

Table 3 Percentage of participants obtaining correct answers on declarative tasks

<b>Content</b>	<b>Declarative knowledge</b>	<b>Low</b>	<b>Moderate</b>	<b>High</b>	<b>Total</b>
Angles	Angle formation concept	12.0	87.6	.4	100
Triangles	Triangle and its properties	9.8	88.1	2.1	100
Quadrilaterals	Recognizing square and its unique properties	16.4	80.4	3.2	100
Prisms	Recognizing triangular prism and its faces	14.3	85.6	.1	100
Pyramids	Naming of one unique property of pyramid and faces of rectangular pyramid	11.2	88.6	.2	100
<b>Average</b>		<b>12.9</b>	<b>86.0</b>	<b>1.1</b>	<b>100</b>

Score ranges: Low = 0-3; Moderate = 4-7; High = 8-11

In this study, declarative knowledge refers to the ability to form angles, describe properties of a given triangle, recognize a given square and its unique properties, recognize triangular prism and its faces and name property of pyramid and faces of rectangular pyramid. The result in Table 3 shows that more than 80% of the participants demonstrated moderate declarative knowledge on angle formation, triangular properties, uniqueness of family of quadrilaterals, angular prism properties and pyramids. Only a few (less than 5%) of the participants demonstrated high declarative knowledge in all the five content areas. The overall result shows that majority of participants exhibited moderate declarative knowledge on shape and space.

Procedural knowledge in this study comprised knowing how to use procedures, sketch shapes and find areas and volumes of shapes. The result of analysis of participants' levels of knowledge is shown in Table 5.

Table 4 Percentage (%) of participants obtaining correct answers on procedural tasks

<b>Content</b>	<b>Procedural Knowledge</b>	<b>Low</b>	<b>Moderate</b>	<b>High</b>	<b>Total</b>
Angles	Sketching angles formed when two straight lines cross each other	16.8	27.6	55.6	100
Triangles	Finding area of right-angled triangle	44.8	5.3	49.9	100
	Finding area of kite	12.9	34.9	52.2	100
	Sketching lines of symmetry	52.6	44.4	3.0	100
Quadrilaterals	Finding area of transformed square(rectangle)	77.3	19.7	3.0	100
	Sketching net/finding area of rectangular prism	35.4	50.3	14.3	100
Prisms	Finding volume of milo tin (cylindrical)	16.8	81.6	1.6	100
Pyramids	Finding volume of cone	54.0	32.9	13.1	100
<b>Average</b>		<b>38.8</b>	<b>37.1</b>	<b>24.1</b>	<b>100</b>

Score ranges: Low = 0-3; Moderate = 4-7; High = 8-11

As shown in Table 4, about 50-56% of participants demonstrated high procedural knowledge on angles, areas of right-angled triangle and kite. However, when the area of square was transformed into rectangle, more than 50% of the participants were unable to solve the task completely. A large proportion (50-82%) of participants also demonstrated moderate procedural knowledge on sketching the net and finding area of a rectangular prism and volume of a cylindrical object. Between 52% and 77% of the participants demonstrated low procedural knowledge of area of triangle, line of symmetry, area of transformed rectangle and volume of cone. The result shows that many of the participants possess moderate procedural knowledge of angles, triangles, quadrilaterals, prisms and pyramids.

For this study, conditional knowledge is viewed as ability to explain why, when and where certain concepts and procedures are related and applied to specific

instances in geometry. Table 5 displays the percentages of participants' conditional knowledge levels.

Table 5 Percentage of participants obtaining correct answers on conditional tasks

<b>Content</b>	<b>Conditional knowledge</b>	<b>Low</b>	<b>Moderate</b>	<b>High</b>	<b>Total</b>
Angles	Corresponding angles and number of angles formed when two straight lines cross each other	12.3	66.7	21.0	100
Triangles	Concept of same angle measures in every isosceles triangle	12.7	64.1	23.2	100
Quadrilaterals	New shape when two opposite sides of square increase by 2 units	15.6	49.4	35.0	100
Prisms	Justifying why whether cylinder is a prism	54.0	45.7	.3	100
Pyramids	Determining new figure formed when triangular prism vertices are pulled to a tip	69.5	25.6	4.9	100
<b>Average</b>		<b>32.8</b>	<b>50.3</b>	<b>16.9</b>	<b>100</b>

Score ranges: Low = 0-3; Moderate = 4-7; High = 8-11

As shown in Table 5, only 23.2% and 35.0% demonstrated high conditional knowledge on isosceles and quadrilaterals. Less than 13% of the participants demonstrated low knowledge in angle measures and in isosceles triangle. Also, as large as 54.0% of participants demonstrated low conditional knowledge on relationship between cylinder and prism or transformation of prisms into pyramids. Overall, results portrayed that, not many participants demonstrated high conditional knowledge on angles, triangles, quadrilaterals, prisms and pyramids. Comparatively, majority demonstrated better knowledge in procedural knowledge and declarative knowledge than in conditional knowledge.

## 2. *Characterization of key learning obstacles in shape and space*

The second research question sought to identify key learning obstacles in preservice teachers' written responses to content knowledge tasks on basic shape and space.

The result of thematic analysis of participants' written responses is displayed in matrix index in Table 6.

Table 6 Matrix index of indicators and type of learning obstacles in shape and space

<i>Indicators to learning obstacles identified</i>	<b>Obstacle type</b>		
	<b>Ont</b>	<b>Epi</b>	<b>Did</b>
<b>Declarative Knowledge</b>			
Angles: Inaccurate generalization of angle formation, e.g. 56.2% said <i>All angles are formed by two or more lines.</i>		√	√
Quadrilaterals: shallow knowledge of lines of symmetry and rotational order of square and rectangle. e.g. 45.7% <i>couldn't state correct number of lines of symmetry and order of rotation of a square.</i>		√	√
Prisms: Poor or no concept image or spatial knowledge of prisms, e.g. 64.9% <i>couldn't name a prism and its faces, when its net is given.</i>	√	√	√
Pyramids: Poor visual abstraction of pyramid, e.g. 45.2% <i>unable to name one unique property of pyramid and faces of rectangular pyramid</i>	√	√	√
<b>Procedural Knowledge</b>			
Angles: Lack appropriate procedures to follow: 87.4% <i>incorrectly sketched of number of angles formed when two straight lines cross each other</i>			√
Quadrilaterals: No/ faulty solution process (incorrect or no procedure applied), e.g. 63.9% <i>couldn't find the area of a transformed square into a rectangle</i>		√	√
Prisms: Lack procedural structure and principle to apply e.g. 65.4% <i>incorrectly sketched of the net and no/wrong solution for area of a rectangular prism</i>	√	√	√
Pyramids: No/incorrect formulae or solution process, e.g. 56.7% <i>couldn't provide complete steps for finding volume of a cone</i>		√	√
<b>Conditional Knowledge</b>			
Angles: Faulty reasoning/generalization or misconception, e.g. 32.3% <i>said corresponding angles are equal no matter the figure</i>		√	√
Triangles: Erroneous generalization or misconception, e.g. 52.7% <i>said every isosceles triangle has same angle measure</i>		√	√
Quadrilaterals: Lack of figural conservation, its configurative and inclusive properties, e.g. 56.7% <i>did determine if a figure (i.e. square) is transformed into a kite, what its area will be</i>		√	√
Prisms: Poor relational knowledge of figural concept and its configurative or inclusive property, e.g. 77.5% <i>unable to justify whether a cylinder is a prism</i>	√	√	√
Pyramids: Lack of figural concept and its configurative or inclusive property, e.g. 76.6% <i>unable to justify whether a cone is a pyramid</i>	√	√	√

*Ont means Ontogenetic; Epi means Epistemological; Did means Didactical*

Three main learning obstacles namely ontogenetic, epistemological and didactical were identified after analyzing the written responses to the tasks eliciting declarative, procedural and conditional knowledge on shape and space. For declarative knowledge in Table 6, 56.2% of the preservice teachers held the notion that *all angles are formed by two or more lines*. They however failed to consider the necessary and sufficient attributes in angle formation such as concept of straight lines. This is characterized as an epistemological obstacle arising from faulty way of thinking, insufficient grasp of facts or poor verbal abstraction of concept of angle. Further examination of the written scripts portrayed didactical obstacle arising from the kind of angular representations and definitions provided by teachers or in textbooks. Also, as shown in Table 6, 45.7% of the preservice teachers did not grasp the concepts and principles of *line of symmetry and rotational order of square and rectangle*. Approximately 65% of the preservice teachers failed to provide *correct naming of prism and its faces when its net was given*. These are clear demonstrations of knowledge gaps or lack of early spatial learning experience in the concepts of symmetry, order or nets of prisms. These obstacles are characterized as ontogenetic and didactic obstacles.

For obstacles related to procedural knowledge as indicated in Table 6, the analysis shows that 87.6%, 63.9%, 65.4% and 56.7 % respectively of the preservice teachers have problems *sketching the angles formed when two straight lines cross each other* (epistemological and didactical), *finding the area of rectangle transformed from a given square* (epistemological and didactical), *sketching net and*

*finding area of rectangular prism* (ontogenetic, epistemological and didactical) as well as *finding the volume of cone* (epistemological and didactical).

Finally, for obstacles related to conditional knowledge, the analysis of the written scripts of preservice teachers shows some difficulties or faulty way of thinking among 32.3% of the preservice teachers on whether *corresponding angles are equal no matter the figure* (epistemological and didactical), . Also, 56.7% of the preservice teachers could not relate their knowledge of conservation to determine the area of a square *if it is transformed into a kite* (epistemological and didactical). About 77%-88% also demonstrated little or no idea of *justifying whether a cylinder is a prism* or *whether a cone is a pyramid* (ontogenetic, epistemological and didactical) using the inclusive-exclusive properties of the shapes.

## **Discussion**

The purpose of this study was to examine the levels of content knowledge and characterize the key learning obstacles in shape and space among preservice teachers preparing for their first practice teaching at basic schools. The study identified three content knowledge types comprising declarative, conditional and procedural knowledge (Schneider & Stein, 2010; Alexander & Judy, 1988) and three key learning obstacles namely ontogenetic, epistemological and didactical. For declarative knowledge, only a few of the participants demonstrated high levels with more than 80% and about 13% demonstrating moderate and low declarative knowledge respectively in angles, triangles, quadrilaterals, prisms and pyramids. This result corroborates the college-wise assessment reports (Institute of Education, 2017; 2014) and furthermore suggests that majority of the participants have

conceptual lapses and lack in-depth knowledge of definitions, properties and representations of shape and space. As underlined in a study by Haj-Yahya and Hershkowitz (2013), limited declarative knowledge of geometric concepts tends to affect the use the necessary and sufficient set of attributes to discriminate shapes. Noto, Priatna and Dahlan (2019) such lapses as learning obstacles arising from epistemological and didactical situations. Addressing these obstacles require the use of model based instructions which will allow preservice teachers to rethink about spatial notions while investigating the inclusive-exclusive properties, definitions and principles underlining shape and space. The practice where tutors of preservice teachers tend to skip or rush to complete lessons on Shape and space in the syllabus, perhaps, because they seem so basic to teach or learn at college level, needs to be averted.

A substantial number of the preservice teachers demonstrated high procedural knowledge in angles, triangles, quadrilaterals, prisms and pyramids. This shows that many of these preservice teachers in the present study effectively applied procedures, formulas or algorithms to solve the problems relating to shape and space. However, their solution processes were affected by their weak declarative and conditional knowledge. This result is consistent with findings by Haapalaso and Kadjevich (2000); and Star and Stylianides (2013) where many students tend to be fluent in applying procedures without meaningful understanding of concepts and their relationships. This knowledge gap often arises from a didactical conceptual situation emphasizes formulae rather than concept definitions, concept images and figural relationships. In developing procedural knowledge, Aydin and Ubuz (2010)

suggests that teachers should relate the problem situation to real life context and allow for independent thinking and social interactions among learners. This has the tendency of generating learners' mental schemas and action-process abilities to tackle both routine and non-routine problems.

Conditional knowledge defines the *why*, *when* and *where* aspects of conceptual knowing in mathematics (Lauritzen, 2012) or geometry (Aydin & Uzbu, 2010). It portrays the depth of an individual's conceptual and procedural knowledge relationships. The present study shows that only 15% of the preservice teachers demonstrated high conditional tasks knowledge on shape and space with majority unable to fully justify or explain conditional statements regarding angles, triangles, quadrilateral, prism and pyramids. This type of obstacle relates to the epistemology of geometry and perhaps poor didactical conceptual structures of geometry content (Fuadiah, 2015). The inability to justify or the faulty thinking demonstrated by preservice teachers may also stem from limited exposure to geometric material or direct teaching practice which fail to link learning situation to real life.

In this study, declarative, conditional and procedural tasks relating to prisms and pyramids appeared more difficult to perform by preservice teachers than tasks on quadrilaterals, triangles and angles. Majority of the preservice teachers have difficulty identifying the name of the given prism (declarative knowledge), in determining the faces in the prism (conditional knowledge) or even in sketching rectangular prism (procedural knowledge) and deriving its area (conditional knowledge). Since understanding solid shapes such as prisms and pyramids forms the basis for advances in science, technology, engineering and mathematics

education (STEM) (Riastuti, Mardiyana & Pramudya, 2017), preservice teachers' poor knowledge can affect their competence to prepare pupils towards future STEM carriers. A renewed focus on 3-D competence development of preservice teachers is needed to address the main objective of teaching shape and space which is largely to develop spatial abilities of pupils. The teaching of shape and space in colleges of education should focus on integrating the “what”, the “how and the “why” aspects of content knowledge of shape and space to enable preservice teachers acquire adequate didactic conceptual structures for teaching.

### **Conclusion**

The study concludes that majority of preservice teachers at the colleges of education do not have substantial declarative, procedural and conditional content knowledge on shape and space to the point where they can teach spatial notions effectively to basic school children. In board terms, characterizing learning obstacles in shape and space into didactical, epistemological and ontogenetic types gives room for making appropriate pedagogical decisions. Such decisions must focus on building preservice teachers' didactic conceptual structures, before they exit into the teaching field. The study thus recommends that tutors should use hands-on activity, origami or dynamic geometry software to teach preservice teachers to integrate their declarative, procedural and conditional knowledge on shape and space. While stressing on the didactic conceptual structures in shape and space, tutors should also endeavor to relate their instruction to the epistemology of geometry of shape and space and the van Hiele's thinking levels for preservice teachers to understand the mental structures, history, development and application of planar and spatial Geometry.

This study was limited to content knowledge and did not involve pedagogical knowledge development in preservice teachers' training programme. Future studies could also examine the multi-dimensional constituents of pedagogical knowledge of preservice teachers to determine the teacher training effect of the college course on shape and space and other aspects of mathematics.

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