Original Research Article

Dynamics of the optical pulse in a nonlinear medium: approach of moment method coupled with the fourth order Runge-Kutta method

Abstract

In this paper, we use the moment method approach to investigate the evolution of pulse parameters in nonlinear medium. The pulse propagation is modelled by the nonlinear Schrödinger equation. The application of moment method leads to variational equations that is be integrated numerically by the fourth order Runge-Kutta method. The results obtained show the variations of some important parameters of the pulse namely the energy, the pulse position, the frequency shift, the chirp and the width.

Keywords: moment method, nonlinear Schrödinger equation.

Introduction

The generalized nonlinear Schrödinger equation (GNLSE) as a nonlinear model has been studied due to its importance in many fields of physics such a nonlinear optics, plasma physics, superconductivity, quantum mechanics (Cao and Zhang (2013); Jayanthi (2004); Agrawal (2001); Kodama and Hasegawa (1987); Mamyshev P V. Chernikov (1990); Agrawal (1997, 2002, 2007); Lin et al. (2007); Hasegawa and Tappert (1973); Konar and Sengupta (1994); Yano and Kon (1980)).

In the general case, it is very difficult to find analytic solution for this equation [Cao et al. (2010)]. With development of soliton theory and computer algebraic system like mathematics, much research papers has been devoted to exact solution of no linear evolution equations, especially travelling wave solution (Cao and Zhang (2013)). Various effective method of searching for exact solution to GNSE have been presented in the literature: the inverse scattering method, the Blacklund transformation, the Adomian method, homothopy perturbation method, the Hirota bilinear method, the Lie group method, the variable separation method, the variational iteration, the Jacobi elliptic function, the expansion method, the auxiliary equation method, the trial function method, the moment method (Hasegawa and Tappert (1973); Konar and Sengupta (1994); Yano and Kon (1980); Belanger (1996); Biazar and Ghazvini (2008); Gorji et al. (2007); He; Sadighi and Ganji (2007); He and Wu (2007); Sweilam (2007); Sweilam and Al-bar (2007); Turget and Ahmet (2007); Edah et al. (2014)). Among them, the moment method is one of the methods which allow to investigate the variations of the fundamental parameters of the system (Jayanthi (2004); Victor et al. (2007); Vlasov et al. (1971); Bryana (2010)). The aim of this paper is to apply the moment method to find a solitary wave solution for GNLSE which is straight forward and concise. The outline of the present paper is as follows . In section 1, we give the mathematical model. In section 2 we solve the problem by variation moment method. In section 3, we use a Gaussian function, we obtain the variational equations of the pulse parameters which are solved by the fourth order of Runge Kutta numerical method. In section 4, we present results and discussions. Finally, we point out the concluding remarks.

1 Mathematical model

The generalized nonlinear Schrödinger equation in the dimensionless form reads Guanmao and Xiaoping (2007)

$$\frac{i\partial\psi}{\partial z} + a\frac{\partial^2\psi}{\partial t^2} + ib\frac{\partial^3\psi}{\partial t^3} + c|\psi|^2\psi - i\frac{c}{\omega_0}\frac{\partial}{\partial t}(|\psi|^2\psi) = 0 \tag{1.1}$$

where $\psi = \psi(z,t)$ is the envelop of the pulse, $z \in [0, L], L > 0$ is the length of the fiber and $t \in \mathbb{R}$ is the time, a the second order of dispersion, b the third order of dispersion, c the coefficient of self-modulation; $\frac{c}{\omega_0}$ represents the self-steepening term. This equation holds for pulses that contain just a few optical cycles where higher nonlinear terms are included, with initial conditions $\psi(z = 0, t)$.

The parameters a, b, c are related to β_2, β_3 as:
$a = -\frac{\beta_2}{2}$, $b = -\frac{\beta_3}{6}$, $c = \bar{\gamma}$

2 Solving the problem by variational moment method

The basic idea of moment method is to treat the optical pulse like a particle whose energy E, position T, the frequency Ω, the root mean square (RMS) σ and the moment related to the chirp of the pulse are defined as Jayanthi (2004); Paré (2011); Ablowitz and Clarkson (1991):

$$E = \int_{-\infty}^{+\infty} |\psi|^2 dt \quad ; \tag{2.1}$$

$$T = \frac{1}{E}\int_{-\infty}^{+\infty} t|\psi|^2 dt \quad ; \tag{2.2}$$

$$\Omega = \frac{i}{2E}\int_{-\infty}^{+\infty} (\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}) dt \quad ; \tag{2.3}$$

$$\sigma^2 = \frac{1}{E}\int_{-\infty}^{+\infty} (t-T)^2|\psi|^2 dt \quad ; \tag{2.4}$$

$$\tilde{C} = \frac{i}{2E}\int_{-\infty}^{+\infty} (t-T)(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}) dt. \tag{2.5}$$

Obviously, the evolution of these pulse parameters depend on the evolution on the pulse itself in the fiber which is governed by the NLS equation (1.1).To find the evolution of these pulse parameters, we use the equations (2.1) to (2.5) along with equation (1.1)

2.1 Energy evolution

Differentiating (2.1) with respect to z, we have:

$$\frac{dE}{dz} = \int_{-\infty}^{+\infty} (\psi^* \frac{\partial \psi}{\partial z} + \psi \frac{\partial \psi^*}{\partial z}) dt. \tag{2.6}$$

Using (1.1) we find that:

$$\frac{\partial \psi}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 \psi}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 \psi}{\partial t^3} - \frac{\bar{\gamma}}{\omega_0}\frac{\partial}{\partial t}(|\psi|^2\psi) + i\gamma|\psi|^2\psi \tag{2.7}$$

After performing calculations, we have:

$$\begin{aligned} \frac{dE}{dz} &= \int_{-\infty}^{+\infty} i\frac{\beta_2}{2}\Big(\psi\frac{\partial^2 \psi^*}{\partial z^2} - \psi^*\frac{\partial^2 \psi}{\partial z^2}\Big) dt + \\ &\quad \int_{-\infty}^{+\infty} \frac{\beta_3}{6}\Big(\psi\frac{\partial^3 \psi^*}{\partial z^3} - \psi^*\frac{\partial^3 \psi}{\partial z^3}\Big) dt - \\ &\quad \frac{\bar{\gamma}}{\omega_0}\int_{-\infty}^{+\infty} \Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^2\psi\Big) + \psi\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\Big] dt \\ &= 0 \end{aligned} \tag{2.8}$$

2.2 Evolution of pulse position

Differentiating (2.2) with respect to z we get:

$$\frac{dT}{dz} = \frac{1}{E}\int_{-\infty}^{+\infty} t\Big(\psi^*\frac{\partial \psi}{\partial z} + \psi\frac{\partial \psi^*}{\partial z}\Big) dt \tag{2.9}$$

we get:

$$\frac{dT}{dz} = i\frac{\beta_2}{2E}\int_{-\infty}^{+\infty} t\Big(\psi\frac{\partial^2\psi^*}{\partial z^2} - \psi^*\frac{\partial^2\psi}{\partial z^2}\Big)dt + \\ \frac{\beta_3}{6}\int_{-\infty}^{+\infty} t\Big(\psi\frac{\partial^3\psi^*}{\partial z^3} - \psi^*\frac{\partial^3\psi}{\partial z^3}\Big)dt - \\ \frac{\bar{\gamma}}{\omega_0 E}\int_{-\infty}^{+\infty} t\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^2\psi\Big) + \psi\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\Big]dt \tag{2.10}$$

After integrating by parts and the definition of frequency in (2.3), we obtain:

$$\frac{dT}{dz} = \beta_2\Omega + \frac{\beta_3}{2E}\int_{-\infty}^{+\infty} t\Big|\frac{\partial\psi}{\partial t}\Big|^2 dt - \frac{3\bar{\gamma}}{2\omega_0 E}\int_{-\infty}^{+\infty} |\psi|^4 dt \tag{2.11}$$

2.3 Evolution of frequency schift

Differentiating (2.3) with respect to z, we have:

$$\frac{d\Omega}{dz} = \frac{i}{2E}\int_{-\infty}^{+\infty}\Big[\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial z}\Big) - \frac{\partial}{\partial z}\Big(\psi\frac{\partial\psi^*}{\partial z}\Big)\Big]dt \tag{2.12}$$

$$\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial z}\Big) = \psi^*\frac{\partial^2\psi}{\partial z\partial t} + \frac{\partial\psi^*}{\partial z}\frac{\partial\psi}{\partial t} \tag{2.13}$$

From (2.7), we can write:

$$\psi^*\frac{\partial^2\psi}{\partial z\partial t} = \frac{i}{2}\psi^*\frac{\partial^2\psi}{\partial t^2} - a\psi^*\frac{\partial^4\psi}{\partial t^4} + ib|\psi|^2\frac{\partial}{\partial t}(|\psi|^2) + \\ ib\psi^*|\psi|^2\frac{\partial\psi}{\partial t} - c|\psi|^2\frac{\partial^2}{\partial t^2}(|\psi|^2) - c\psi^*|\psi|^2\frac{\partial^2\psi}{\partial t^2} \tag{2.14}$$

and

$$\frac{\partial\psi^*}{\partial z}\frac{\partial\psi}{\partial t} = -\frac{i}{2}\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t} - a\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t} - ib|\psi|^2\psi^*\frac{\partial\psi}{\partial t} \\ -c\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\frac{\partial\psi}{\partial t} \tag{2.15}$$

Adding (2.14) and (2.15) and substituting into (2.13), we find:

$$\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial z}\Big) = \frac{i}{2}\left[\psi^*\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t}\right] - a\left[\psi^*\frac{\partial^4\psi}{\partial t^4} + \right. \\ \left.\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t}\right] + ib|\psi|^2\left[\frac{\partial}{\partial t}(|\psi|^2) + \psi^*\frac{\partial\psi}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\right] - \\ c|\psi|^2\left[\frac{\partial^2}{\partial t^2}(|\psi|^2) + \psi^*\frac{\partial^2\psi}{\partial t^2} + \Big|\frac{\partial\psi}{\partial t}\Big|^2\right] - \\ c\psi^*\frac{\partial}{\partial t}(|\psi|^2)\frac{\partial\psi}{\partial t} \tag{2.16}$$

Also, we can write

$$\frac{\partial}{\partial z}\Big(\psi\frac{\partial\psi^*}{\partial z}\Big) = -\frac{i}{2}\Bigg[\psi\frac{\partial^2\psi^*}{\partial t^2} - \frac{\partial^2\psi}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Bigg] - a\Bigg[\psi\frac{\partial^4\psi^*}{\partial t^4} + \frac{\partial^3\psi}{\partial t^3}\frac{\partial\psi^*}{\partial t}\Bigg] + ib|\psi|^2\Bigg[\frac{\partial}{\partial t}(|\psi|^2) + \psi\frac{\partial\psi^*}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\Bigg] - c|\psi|^2\Bigg[\frac{\partial^2}{\partial t^2}(|\psi|^2) + \psi\frac{\partial^2\psi^*}{\partial t^2} + \Big|\frac{\partial\psi}{\partial t}\Big|^2\Bigg] - c\psi\frac{\partial}{\partial t}(|\psi|^2)\frac{\partial\psi^*}{\partial t} \tag{2.17}$$

Using (2.16) and (2.17) into (2.12), we can find the evolution of frequency along the fiber to be

$$\frac{d\Omega}{dt} = \frac{i}{2E}\int_{-\infty}^{+\infty}\frac{i\beta_2}{2}\Bigg[\Big(\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t} + \frac{\partial\psi^2}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Big) - \Big(\psi\frac{\partial^3\psi^*}{\partial t^3} + \psi^*\frac{\partial^3\psi}{\partial t^3}\Big)\Bigg]dt + \frac{i}{2E}\int_{-\infty}^{+\infty}\frac{\beta_3}{6}\Bigg[\Big(\psi^*\frac{\partial^4\psi}{\partial t^4} - \psi\frac{\partial^4\psi^*}{\partial t^4}\Big) + \Big(\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t} - \frac{\partial\psi^3}{\partial t^3}\frac{\partial\psi^*}{\partial t}\Big)\Bigg]dt - \frac{i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}|\psi|^2\Big(\psi^*\frac{\partial^2\psi}{\partial t^2} - \psi\frac{\partial^2\psi}{\partial t^2}\Big)dt + -\frac{3i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi}{\partial t}\Big)dt - \frac{\bar{\gamma}}{E}\int_{-\infty}^{+\infty}|\psi|^2\frac{\partial}{\partial t}|\psi|^2 dt \tag{2.18}$$

In order to calculate $\frac{d\Omega}{dz}$, we evaluate one by one the integrals on right hand side of the (2.18). After computation, we get:

$$\frac{d\Omega}{dt} = -\frac{i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}|\psi|^2\Big(\psi^*\frac{\partial^2\psi}{\partial t^2} - \psi\frac{\partial^2\psi}{\partial t^2}\Big)dt - \frac{3i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi}{\partial t}\Big)dt - \frac{\bar{\gamma}}{E}\int_{-\infty}^{+\infty}|\psi|^2\frac{\partial}{\partial t}|\psi|^2 dt \tag{2.19}$$

Rearanging (2.19) and after computation, we obtain

$$\frac{d\Omega}{dt} = -\frac{i\bar{\gamma}}{E\omega_0}\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi}{\partial t}\Big)dt \tag{2.20}$$

2.4 Evolution of chirp parameter

Let's differentiate (2.5) with respect to z, we have:

$$\frac{d\tilde{C}}{dz} = \frac{i}{2E}\int_{-\infty}^{+\infty}(t-T)\Big[\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial z}\Big) - \frac{\partial}{\partial z}\Big(\psi\frac{\partial\psi^*}{\partial z}\Big)\Big]dt \tag{2.21}$$

From (2.16) and (2.17), we have:

$$\begin{aligned}
&\frac{d\tilde{C}}{dz} = \frac{\beta_2}{4E}\int_{-\infty}^{+\infty}(t-T)\Bigg[\Big(\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t} + \frac{\partial\psi^2}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Big) - \\
&\Big(\psi\frac{\partial^3\psi^*}{\partial t^3} + \psi^*\frac{\partial^3\psi}{\partial t^3} + \Big)\Bigg]dt \\
&\frac{i}{2E}\int_{-\infty}^{+\infty}(t-T)\frac{\beta_3}{6}\Bigg[\Big(\psi^*\frac{\partial^4\psi}{\partial t^4} - \psi\frac{\partial^4\psi^*}{\partial t^4}\Big) + \\
&\Big(\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t} - \frac{\partial\psi^3}{\partial t^3}\frac{\partial\psi^*}{\partial t}\Big)\Bigg]dt - \\
&\frac{i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}(t-T)|\psi|^2\Big(\psi^*\frac{\partial^2\psi}{\partial t^2} - \psi\frac{\partial^2\psi}{\partial t^2}\Big)dt - \\
&\frac{3i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}(t-T)\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi}{\partial t}\Big)dt - \\
&\frac{\bar{\gamma}}{E}\int_{-\infty}^{+\infty}(t-T)|\psi|^2\frac{\partial}{\partial t}|\psi|^2 dt
\end{aligned} \tag{2.22}$$

After many integrations by parts, we get:

$$\begin{aligned}
&\frac{d\tilde{C}}{dz} = \frac{\beta_2}{E}\int_{-\infty}^{+\infty}\Big|\frac{\partial\psi}{\partial t}\Big|^2 dt + \frac{i\beta_3}{4E}\int_{-\infty}^{+\infty}\Bigg[\Big(\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t} - \\
&\frac{\partial\psi^2}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Big)dt - \frac{i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}|\psi|^2\Big(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t} - \Big)dt \\
&\frac{3i\bar{\gamma}}{2E\omega_0}\int_{-\infty}^{+\infty}(t-T)\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi}{\partial t}\Big)dt + \\
&\frac{\bar{\gamma}}{2E}\int_{-\infty}^{+\infty}\Big|\psi\Big|^4 dt
\end{aligned} \tag{2.23}$$

2.5 Evolution of the RMS width

We differentiate (2.4) with respect to z to obtain

$$2\sigma E\frac{d\sigma}{dz} = \int_{-\infty}^{+\infty}(t-T)^2(\psi^*\frac{\partial\psi}{\partial z} + \psi\frac{\partial\psi^*}{\partial z})dt \tag{2.24}$$

$$\begin{aligned}
&2\sigma E\frac{d\sigma}{dz} = i\frac{\beta_2}{2}\int_{-\infty}^{+\infty}(t-T)^2\Big(\psi\frac{\partial^2\psi^*}{\partial t^2} - \psi^*\frac{\partial^2\psi}{\partial t^2}\Big)dt - \\
&\frac{\bar{\gamma}}{\omega_0}\int_{-\infty}^{+\infty}(t-T)^2\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^2\psi\Big) + \psi\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\Big]dt + \\
&\frac{\beta_3}{6}\int_{-\infty}^{+\infty}(t-T)^2\Big(\psi^*\frac{\partial^3\psi}{\partial t^3} + \psi^*\frac{\partial^3\psi^*}{\partial t^3}\Big)dt
\end{aligned} \tag{2.25}$$

$$\frac{d\sigma}{dz} = \frac{\beta_2 c}{\sigma} + \frac{\beta_3}{2\Gamma E}\int_{-\infty}^{+\infty}(t-T)\left|\frac{\partial\psi}{\partial t}\right|^2 dt \tag{2.26}$$

3 Numerical simulation with Runge-Kutta 4

Let's choose the pulse shape on the Gaussian form **?**Fisher and Bischel (1975):

$$\psi(z,t) = A\exp\left[i\varphi - i\Omega(t-T) - (1+iC)\frac{(t-T)^2}{2\tau^2}\right] \tag{3.1}$$

with $\tau^2 = K\sigma^2$, $C = 2\tilde{C}$, $K = cte$

We obtain a variational equations for each parameter as follows:

$$\begin{aligned}
\frac{dE}{dz} &= 0\ ; \\
\frac{dT}{dz} &= \beta_2\Omega + \beta_3/2\left(\Omega^2 + \frac{1+C^2}{2\tau^2}\right) + \frac{3\bar{\gamma}E}{\sqrt{8\pi}\omega_0\tau}\ ; \\
\frac{d\Omega}{dz} &= \frac{\bar{\gamma}EC}{\sqrt{2\pi}\omega_0\tau^3}\ ; \\
\frac{dC}{dz} &= 2\beta_2\Omega^2 + \beta_2\frac{1+C^2}{\tau^2} + \beta_3\frac{1+C^2}{2\tau^2} + \frac{4\bar{\gamma}E\Omega}{\sqrt{2\pi}\omega_0\tau} + \frac{\bar{\gamma}E}{\sqrt{2\pi}\tau}\ ; \\
\frac{\tau}{dz} &= \frac{\beta_2 C}{\tau} + \frac{\beta_3\Omega C}{\tau}.
\end{aligned}$$

We solve the variational equations by the fourth order of Runge Kutta method Butcher.

4 Discussion

$\frac{dE}{dz} = 0$, therefore the pulse energy remains constant when the pulse propagates along the fiber. Since the width increases (fig. 5) then the pulse flattens and according to equation (2.26), this flattening is due to dispersion effects. The increasing of the chirp confirm this. The equation (2.11) shows that the pulse position is affected by any frequency shift due to the group velocity dispersion β_2 and the third order dispersion β_3. As the pulse propagates, the frequency increases quickly at the beginning then reach a limit value after a distance (fig.2).

The equation (2.21) shows that the chirp is not affected by the group velocity dispersion β_2 nor the self-steepening $\bar{\gamma}$. The equation (2.26) shows that the evolution of the width depends on the group velocity dispersion β_2 and the third order dispersion β_3; it's not affected by the self-steepening parameter.

The above equations for the evolution of the pulse parameter reduce the complexity of the problem but they are still not a useful form because they depend on the shape $\psi(z,t)$, which is not known until 1.1 is solved. If one has some knowledge of the pulse shape and its dependence on the five moments, the problem can be solved approximately.

Conclusion

To illustrate results of moment method for ultra short pulse propagation in optical fibers, we have carried out numerical investigation on the evolution of pulse parameters along the propagation direction z. So, we have used standard fourth order of Runge-Kutta method to integrate the system of ordinary differential equations obtained by the application of moment method.

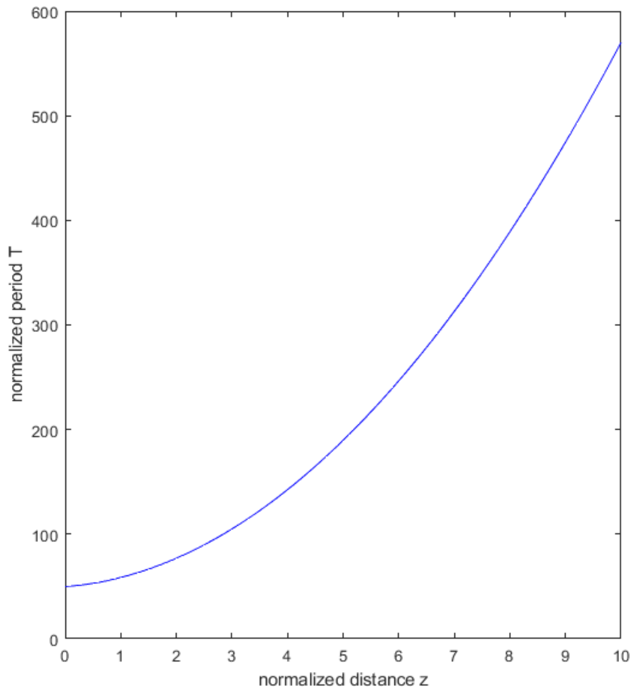

Figure 1: Variations of the period T with respect to the distance z

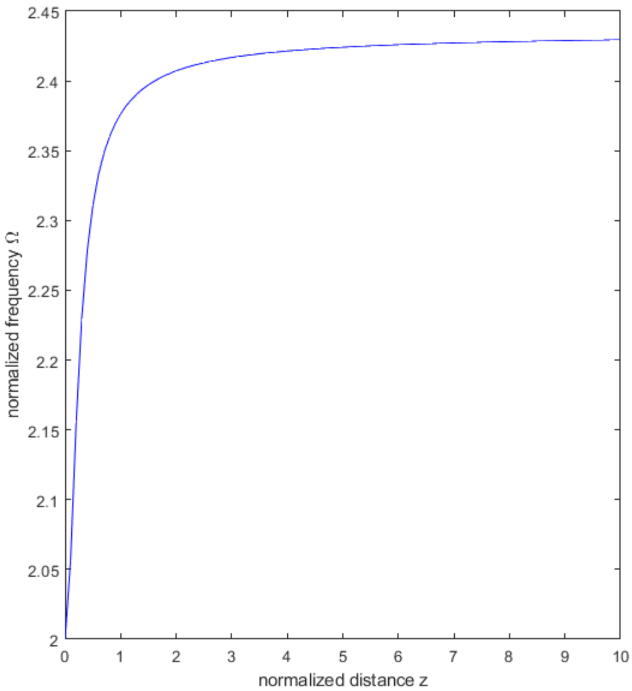

Figure 2: Variation of the frequency Ω with respect to the distance z

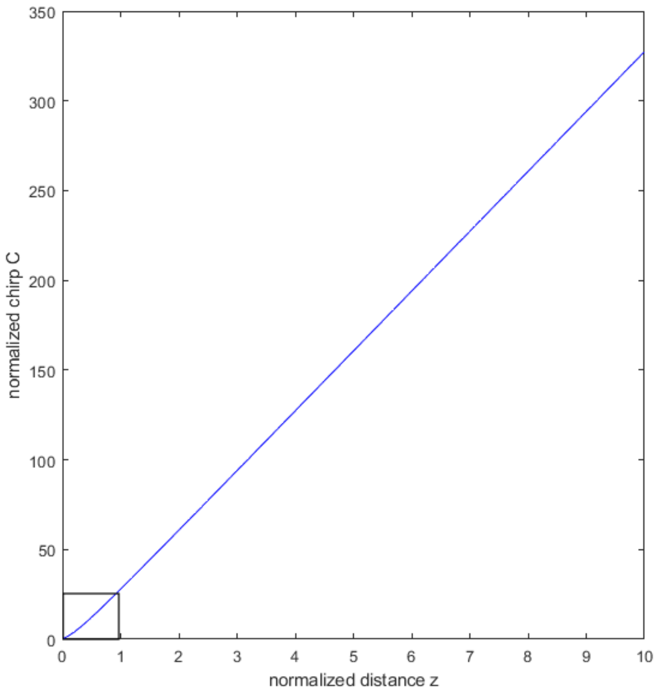

Figure 3: Variation of the chirp C with respect to the distance z

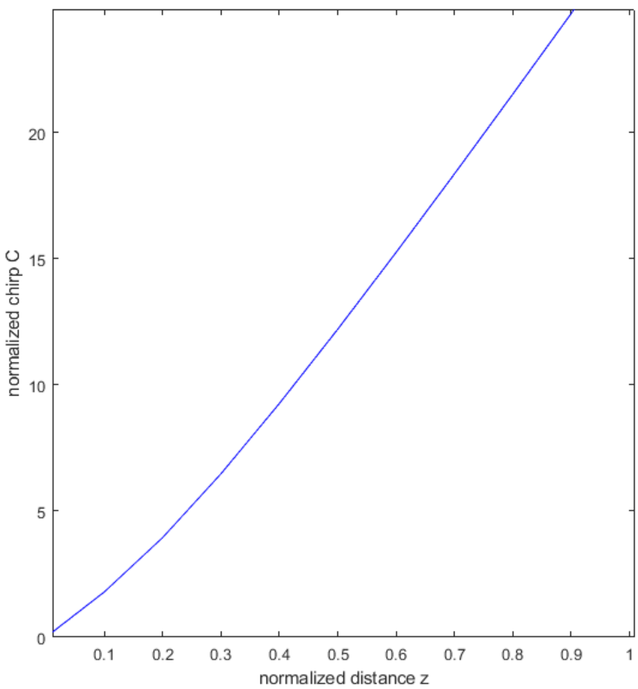

Figure 4: Zoom of the framed part of the chirp C

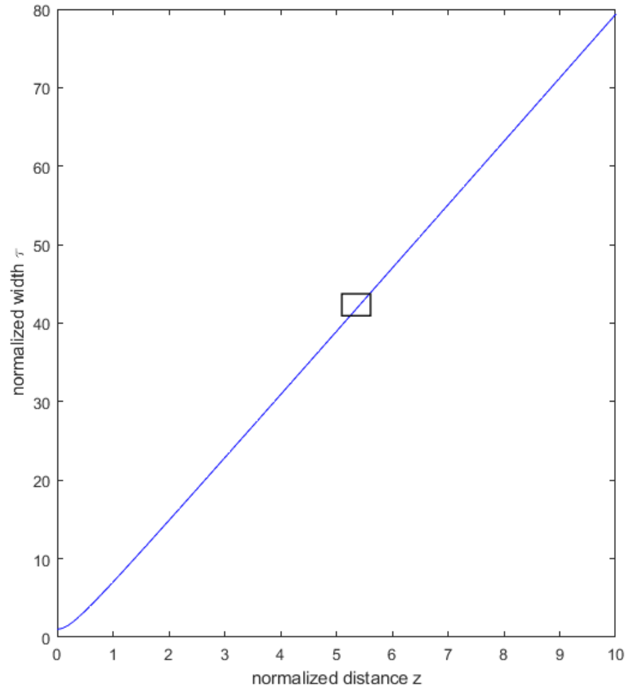

Figure 5: Variation of the width τ with respect to the distance z

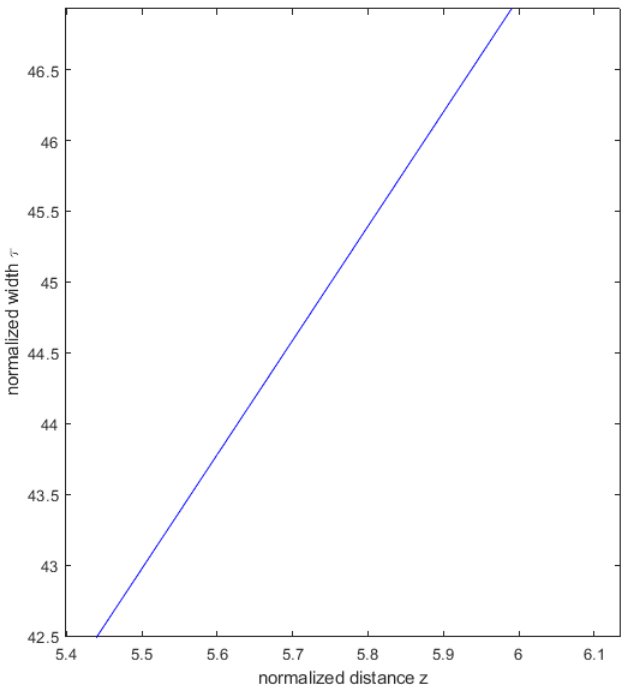

Figure 6: Zoom of the framed part of the width τ

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