Common Neighbourhood and Common Neighbourhood Domination in fuzzy Graphs

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Abstract

In this paper the concepts of common neighbourhood and common neighbourhood domination in fuzzy graph G was introduced and investigated and denoted by N_{cn} and γ_{cn}. We obtained many results related to $\gamma_{cn}(G)$ and γ_{cn}. Finally we give the relationship of $\gamma_{cn}(G)$ with some other parameters in fuzzy graphs.

Keywords:- fuzzy graph common-neighbourhood, common-neighbourhood domination number and Inj-neigborhood.

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1 Introduction

In the last 60 years, Graph theory has seen an explosive growth due to interaction with areas like computer science, electrical and communication engineering, Operations Research etc. In (2011) A. Alwardi, N.D Soner and Karam Ebadi [1] introduced and studied common neighborhood dominating set $CN-domintion$, after two year A. Alwardi and N. D. Soner [2] introduced and investigated the concept of common neighbourhood edge dominating set $CN-edge$ domination, all the graph considered here are finite and undirected with no loops and multiple edges. In (2017) P. Dunder, A. Aytac and E. Kilic [8]introduced and investigated the concept of common neighborhood $CN-neighbourhood$ [8] after one year (Asma et. al.) introduced and investigsted on common neighbourhood graph [3]. In (1973), Kaufmann [4] introduced definition of fuzzy graphs. Rosenfeld [5] introduced another elaborated definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness, etc. Perhaps the fastest growing area within graph and fuzzy graph is the study of domination, the reason being its many and varied applications in such fields as social sciences, communication networks, algorithm designs, computational complexity etc. There are several types of domination depending upon the nature of domination which motivated us to introduce the concepts common neighborhood $CN-neighbourhood$ and the concept of common neighborhood dominating set also common neighbourhood domination number CN- domination number γ_{cn} in fuzzy graph. The concept of domination in fuzzy graphs was investigated by Somasundaram and Somasundaram [6] and A. Somasundaram [7]. In this paper we introduce the concept of concepts common neighborhood $CN-neighbourhood$ and the concept of common neighborhood dominating set and $CN-domination$ number in fuzzy graphs using effective edges. we obtain some interesting results for this Parameter in fuzzy graphs.

2 Preliminaries

In this section we review some basic definitions related to common neighbourhood and common neighbourhood domination of graphs, also basic definiations related to fuzzy graphs and domination in fuzzy graphs.

Let G be simple graph with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$. For $i \neq j$, the common neighborhood of the vertices v_i and v_j, denoted by $\Gamma(v_i, v_j)$, is the set of vertices, different from v_i and v_j, which are adjacent to both v_i andv_j. Let $G = (V, E)$. For any vertex $u \in V$ the CN-neighbourhood of u denoted by $N_{cn}(u)$ is defined as $N_{cn}(u) = \{v \in N(u) : |\Gamma(u, v)| \geq 1\}$. The cardinality of $N_{cn}(u)$ is called the common neighbourhood degree $CN - degree$ of $u$and denoted by $deg_{cn}(u)$ in G, and $N_{cn}[u] = N_{cn}(u) \cup \{u\}$. The maximum and minimum common neighbourhood degree of a vertex in G are denoted respectively by $\Delta_{cn}(G)$ and $\delta_{cn}(G)$. That is $\Delta_{cn}(G) = max\ u \in V|N_{cn}(u)|$ and $\delta_{cn}(G) = min\ u \in V|N_{cn}(u)|$. If u and v are any two adjacent vertices in V such that $|\Gamma(u, v)| \geq 1$, then we say u is common neighbourhood adjacent CN-adjacent to v or u is CN-dominate v. Let $G = (V, E)$ be a graph and $u \in V$ such that $|\Gamma(u, v)| = 0$ for all $v \in N(u)$. Then u is in every common neighbourhood dominating set, such points are called common neighbourhood isolated vertices. Let I_{cn} denote the set of all common neighbourhood isolated vertices of G. Hence $I_s \subseteq I_{cn} \subseteq D$, where I_s is the set of isolated vertices and D is the minimum $CN - dominating$ set of G. A subset S of V is called a common neighbourhood independent set (CN-independent set), if for every $u \in S; v \notin N_{cn}(u)$ for all $v \in S - \{u\}$. It is clear that every independent set is CN -independent set. The CN-independent set S is called maximal if any vertex set properly containing S is not CN-independent set. The maximum cardinality of CN-independent set is called common neighbourhood independence number (CN-independence number) and denoted by β_{cn}, and the lower CN-independence number i_{cn} is the minimum cardinality of the CN-maximal independent set. Let $G = (V, E)$ A subset S of V is called Common neighbourhood vertex covering

CN-vertex covering of G if for any CN-edge $e = uv$ either $u \in S$ or $v \in S$. The minimum cordiality of CN-vertex covering of G is called the CN-covering number of G and denoted by $\alpha_{cn}(G)$. Let $G = (V, E)$ be a graph a subset D of V is called common neighbourhood dominating set CN-dominating set if for every vertex $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\Gamma(u, v)| \geq 1$, where $|\Gamma(u, v)|$ is the number of common neighbourhood between the vertices u and v. The common neighbourhood domination number γ_{cn} CN-domination number is the minimum cardinality of a common neighbourhood dominating set of G. A fuzzy graph $G = (V, \mu, \rho)$ is a non-empty set V together with a pair of functions $\mu : V \longrightarrow [0, 1]$ and $\rho : V \times V \longrightarrow [0, 1]$ such that for all $x, y \in V$, $\rho(x, y) \leq \mu(x) \wedge \mu(y)$. We call μ and ρ the fuzzy vertex set and the fuzzy edge set of G, respectively. Let $G = (\mu, \rho)$ be fuzzy graph with the underlying set V, the order of G is defined as $\sum_{v_i \in V} \mu(v_i)$ and is denoted by p. The size of G is defined as $\sum_{(v_i, v_j) \in E} \rho(v_i, v_j)$ and is denoted by q. The maximum degree of G is $\Delta(G) = \vee\{d(v) : v \in V\}$, and the minimum degree of G is $\delta(G) = \wedge\{d(v) : v \in V\}$. Let $G = (\mu, \rho)$ be a fuzzy graph and let $v \in V(G)$. The edge btween any vertices u and V in G is called effective edge if $(\rho(u, v) = \mu(u) \wedge \mu(v))$. The vertex v is adjacent to a vertex u, if they reach between the effective edge. The effevtive degree of vertex $v \in V(G)$ is defined as $d(v) = \sum_{u \neq v} \rho(u, v)$ and is denoted by $d_E(v)$. Two vertices v_i and v_j are said to be neighbors in a fuzzy graph G, Then $N(v) = \{u \in V : \rho(u, v) = \mu(u) \wedge \mu(v)\}$ is called the open neighborhood set of v and $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood set of v. A fuzzy graph $G = (\mu, \rho)$ is siad to be strong fuzzu graph if $\rho(u, v) = \mu(u) \wedge \mu(v)$ for all $(u, v) \in \rho^*$. A complete fuzzy graph is a fuzzy graph $G = (\mu, \rho)$ such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ for all u and v. A fuzzy graph $G = (\mu, \rho)$ is said to be bipartite if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $\rho(u, v) = 0$ if $u, v \in V_1$ or $u, v \in V_2$. Further, if $\rho(u, v) = \mu(u) \wedge \mu(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called complete bipartite fuzzy graph and is denoted by K_{μ_1,μ_2} where μ_1 and μ_2 are, respectively, the restrictions of μ to V_1 and V_2. Let $G = (V, \mu, \rho)$ be a fuzzy graph. Then we call fuzzy vertices $(u, \mu(u))$ and $(v, \mu(v))$ adjacent if and only if $\rho(u, v) = \mu(u) \wedge \mu(v) > 0$. In a fuzzy graph $G = (\mu, \rho)$ a fuzzy vertex and a fuzzy edge are said to be incident if a fuzzy vertex is the end vertex of a fuzzy edge and if they are incident, then they are said to cover each other. For any threshold $t, 0 \leq t \leq 1, \mu_t = \{x \in V : \mu(x) \geq t\}$ and $\rho_t = \{(x, y) \in V \times V : \rho(x, y) \geq t\}$. Since $\rho(x, y) \leq \mu(x) \wedge \mu(y), \forall x, y \in V$ we have $\rho_t \subseteq \mu_t \times \mu_t$, so that (μ_t, ρ_t) is a graph with the vertex set μ_t and edge set ρ_t for all $t \in [0, 1]$. Let $G = (\mu, \rho)$ be a fuzzy graph, if $0 \leq \alpha \leq t \leq 1$, then (μ_t, ρ_t) is a subgraph of $(\mu_\alpha, \rho_\alpha)$. A path p in a fuzzy graph $G = (\mu, \rho)$ is a sequence of distinct vertices $v_0, v_1, v_2, ..., v_n$ (except possibly v_0 and v_n) such that $\mu(v_i) > o,\ \rho(v_{i-1}, v_i) > 0,\ 0 \leq i \leq 1$. Here $n \geq 1$ is called the length of the path p. The consecutive pairs (v_{i-1}, v_i) are called the edges of the path.

Let $G = (\mu, \rho)$ be a fuzzy graph on V. Let $u, v \in V$. We say that u dominates v in G if $\rho(u, v) = \mu(u) \wedge \mu(v)$. A subset D of V is called a dominating set in G if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. The minimum fuzzy cardinality of dominating sets in G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set D of a fuzzy graph G is said to be a minimal dominating set if no proper subset of S is dominating set of G. The maximum fuzzy cardinality of a minimal dominating set is called the upper domination number of G and is denoted by $\Gamma(G)$.

3 The Common Neighbourhood in Fuzzy Graph

Definition 3.1 *Let $G = (\mu, \rho)$ be fuzzy graph with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$. For $i \neq j$, the common neighborhood of the vertices v_i and v_j, denoted by $\Gamma(v_i, v_j)$, is the set of vertices, different from v_i and v_j, which are* *adjacent to both* v_i *and* v_j

Definition 3.2 *Let* $G = (\mu, \rho)$. *be a fuzzy graph for any vertex* $u \in V$ *the CN-neighbourhood of* u *denoted by* $N_{cn}(u)$ *is defined as* $N_{cn}(u) = \{v \in N(u) : |\Gamma(u,v)|>0\}$.

Definition 3.3 *The fuzzy cardinality of* $N_{cn}(u)$ *is called the common neighbourhood degree* $(CN - degree)$ *of* u *and denoted by* $d_{cn}(u)$ *in* G*, and* $N_{cn}[u] = N_{cn}(u) \cup \{u\}$ *is called the closed common neighbourhood degree* $(CN - degree)$ *of* u*. The maximum and minimum common neighbourhood degree of a fuzzy graph* G *are denoted respectively by* $\Delta_{cn}(G)$ *and* $\delta_{cn}(G)$*. That is* $\Delta_{cn}(G) = max\{d_{cn}(u); u \in |N_{cn}(u)|\}$ *and* $\delta_{cn}(G) = min\{d_{cn}(u); u \in |N_{cn}(u)|\}$.

Example 3.4 *consider the fuzzy graph* G *given in the figure* 1.

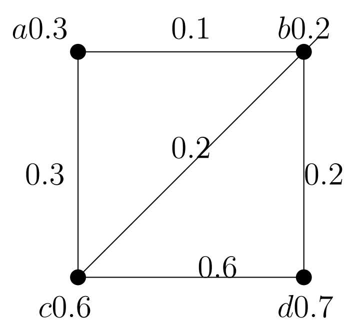


Fig 1.

Then $N_{cn}(a) = \phi$, $N_{cn}(b) = \{c,d\}$, $N_{cn}(c) = \{b,d\}$, $N_{cn}(d) = \{b,c\}$, $deg_{cn}(a) = 0$, $deg_{cn}(b) = 1.3$, $deg_{cn}(c) = 0.9$ *and* $deg_{cn}(d) = 0.8$ *the vertex* a *is* $CN - isolated$ $\Delta_{cn}(G) = 1.3$, *and* $\delta_{cn}(G) = 0.8$

4 Common Neighbourhood Domination in fuzzy Graphs

Definition 4.5 *Let $G = (\mu, \rho)$ be a fuzzy graph and let u and v are any two adjacent vertices in G such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ and $|\Gamma(u, v)| > 0$, then we say u is common neighbourhood adjacent (CN-adjacent) to v or u is CN-dominate v.*

Definition 4.6 *Let $G = (\mu, \rho)$ be a fuzzy graph a subset D of V is called common neighbourhood dominating set $(CN - dominating)$ if for every vertex $v \in V - D$ there exists a vertex $u \in D$, such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ and $|\Gamma(u, v)| > 0$, where $\Gamma(u, v)$ is the number of common neighbourhood btween the vertices u and v, the common neighbourhood domination number $CN - domination$ number is the minimum fuzzy cardinality taken over all minimal common neighbourhood dominating sets of G and is doneted by $\gamma_{cn}(G)$ or γ_{cn}.*

Definition 4.7 *Let $G = (\mu, \rho)$ be a fuzzy graph a common neighbourhood dominating set D is said to be minimal common neighbourhood dominating set if $D - \{u\}$ is not common neighbourhood dominating set of G for all $v \in D$. A minimal common neighbourhood dominating set D is called minimum common neighbourhood dominating set of G if $|D| = \gamma_{cn}(G)$ and is denoted by $\gamma_{cn} - set$.*

Example 4.8 *Consider the fuzzy graph G given in the figure 1.*
We have, $D_{cn1} = \{a, b\}$, $D_{cn2} = \{a, c\}$ and $D_{cn3} = \{a, d\}$ are minimal CN-dominating sets. Then the minimum common neighbourhood number $\gamma_{cn} = min\{|D_{cn1}|, |D_{cn2}|, |D_{cn3}|\} = min\{0.5, 0.9, 1\} = 0.5$.

Theorem 4.9 *A common neighbourhood dominating set D_{cn} of a fuzzy graph G, is minimal common neighbourhood dominating set if and only if one of the folloing condition holds:*

(i). $N_{cn}(u) \cap D_{cn} = \phi$

(ii). There is a vertex $v \in V - D_{cn}$*, such that* $N_{cn}(v) \cap D_{cn} = \{u\}$.

Proof. Let G be a fuzzy graph and let D_{cn} be a minimal common neighbourhood dominating set. Then $D_{cn} - \{v\}$ is not common neighbourhood dominating set. Then there exists a vertex v in $V - D_{cn} - \{v\}$ such that u is not CN-dominated by any vertex of $D_{cn} - \{v\}; u \in V$ if $u = v$, then $N(u) \cap D_{cn} = \phi$, if $u \neq v$, then $N(v) \cap D_{cn} = \{u\}$.

Conversely. Suppose that D_{cn} is CN-dominating set and for each vertex u in D_{cn} one of the two condition holds. Now, we want to prove that D_{cn} is minimal. Suppose D_{cn} is not minimal. Then there exists a vertex $v \in D_{cn}$ such that $D_{cn} - \{v\}$ is CN-dominating set. Thus, u is CN-adjacent to at least one vertex in $D_{cn} - \{v\}$. Hence condition (i) does not hold, also if $D_{cn} - \{v\}$ is CN-dominating set, then every vertex in $V - D_{cn}$ is CN-adjacent to at least one vertex in $D_{cn} - \{v\}$. That means condition (ii) does not hold. So we get contradiction. Hence D_{cn} is minimal common neighbourhood dominating set

Theorem 4.10 *Let G be a fuzzy graph with common neghbourhood isolated vertices if D_{cn} is minimal common neighbourhood dominating set. Then $V - D_{cn}$ is CN-dominatiing set.*

Proof. Let D_{cn} be a minimal CN-dominating set of G.

Suppose that $V - D_{cn}$ is not CN-dominating set. Then there exists a vertex u in D_{cn} such that u is not CN-dominated by any vertex in $V - D_{cn}$. Then u is CN-dominated by at least one vertex v in $D_{cn} - \{u\}$. Thus $D_{cn} - \{u\}$ is common neighbourhood dominating set of G which contradicts the common neighbourhood dominating set of D_{cn}, Then every vertex in D_{cn} is CN-adjacent with at least one vertex in $V - \{D_{cn}\}$. Hence $V - \{D_{cn}\}$ is CN-dominating set.

Theorem 4.11 *For any fuzzy graph G,*

$$\gamma(G) \leq \gamma_{cn}(G)$$

Proof. Since every CN-dominating set of a fuzzy graph G is dominating set of G. Then

$$\gamma(G) \leq \gamma_{cn}(G)$$

In the following we give γ_{cn} for some standard fuzzy graphs,

Proposition 4.12 *For any fuzzy graph G,*

1- If $G = P_n$ is a path. Then $\gamma_{cn}(P_p) = p$.

2- If $G = c_n$ be a cycle fuzzy graph. Then $\gamma_{cn}(C_p) = p$.

3- If $G = K_\mu$ be a complete fuzzy graph. Then $\gamma_{cn}(K_\mu) = min\{\mu(v) : v \in V(K_\mu)\}$.

Theorem 4.13 *For a complete bipartite fuzzy graph K_{μ_1,μ_2} with $|V_1| = p_1$ and $|V_2| = p_2$,*

$$\gamma_{cn}(K_{\mu_1,\mu_2}) = p$$

Proof. Let G be complete bipartite fuzzy graph; Then $\rho(v_1v_2) = 0$ and $\Gamma(v_1v_2) = o$ for all $(v_1v_2) \in V_1$ or V_2 and $\rho(u,v) = \mu(u) \wedge \mu(v), \forall u \in V_1$ and $v \in V_2$. Thus every vetex in V_1 has not common neighbourhood in V_1 also similarly evry vertex in V_2. Hance

$$\gamma_{cn} = p_1 + p_2 = p$$

.

Theorem 4.14 *For any fuzzy graph.*

$$\gamma_{cn}(G) \leq p - \Delta_{cn}(G)$$

Proof. Let $G = (\mu, \rho)$ be any fuzzy graph and let $v \in V(G)$, such that $d_{cn}(v) = \Delta_{cn}(G).\forall u \in N_{cn}(v)$. Then there exsits at least one vertex $w \in V - N_{cn}(v)$ such that $\rho(w,v) = \mu(w) \wedge \mu(v)$ and $|\Gamma(w,v)| > 0$. Thus $V - N_{cn}(v)$ is CN- dominating of G. Hance

$$\gamma_{cn}(G) \leq |V - N_{cn}(v)|$$

$$\gamma_{cn}(G) \leq p - \Delta_{cn}(G)$$

Corollary 4.15 *For any fuzzy graph.*

$$\gamma_{cn}(G) \leq p - \delta_{cn}(G)$$

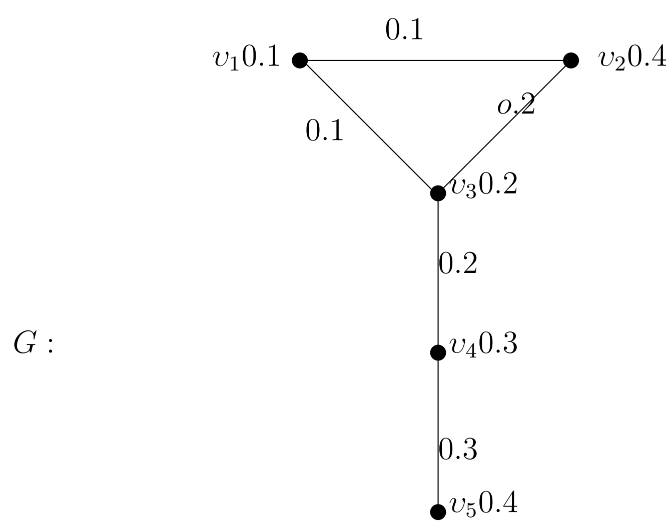
Proof. Since $\delta_{cn} \leq \Delta_{cn}$ and by the above theorm then $\gamma_{cn}(G) \leq p - \delta_{cn}(G)$

Definition 4.16 *Let $G = (\mu, \rho)$ be a fuzzy graph a subset D of V is called common neighbourhood independent set ($CN - independent$) if for every pair of vertices $v, u \in D$ and $u \notin N_{cn}(v)$ and $v \notin N_{cn}(u)$. The maximum fuzzy cardinality teken over all CN- independent sets in a fuzzy graph G is called the CN- independent number of G and is denoted by $\beta_{cn}(G)$ or β_{cn}.*

Definition 4.17 *Let $G = (\mu, \rho)$ be a fuzzy graph a vertex subset S of V is called common neighbourhood vertex covering set ($CN-vertexcovering$) set of G, $CN-$ edge $e = uv$ such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ ethier $u \in S$ or $v \in S$. The minimum fuzzy cardinality teken over all CN- vertex covering sets in a fuzzy graph G is called the CN- vertex covering number of G and is denoted by $\alpha_{cn}(G)$ or α_{cn}.*

Remark 4.18 *If G a fuzzu graph has no $CN - edge$, Then $\alpha_{cn}(G) = 0$*

Example 4.19 *For the fuzzy graph G given in $figure$ 2*



$Fig.$ 2

In figure (2) vertex subsets $\{v_1, v_4, v_5\}, \{v_2, v_4, v_5\}, \{v_3, v_4, v_5\}$ are CN-dominating *sets. Then the minimum fuzzu cardinality of minimal $CN-dominating$ sets is* 0.8*. Hence* $\gamma_{cn} = 0.8$.

The CN-vertex covering set is $\{v_1, v_3\}$. *Then* $\alpha_{cn} = 0.3$.

The maximial $CN-independent$ *set is* $\{v_2, v_4, v_5\}$, *So* $\beta_{cn} = 1.1$

Theorem 4.20 *Let G be a fuzzy graph of order p, Then*

$$\alpha_{cn}(G) + \beta_{cn}(G) = p$$

Proof. Let S be CN-independent set in G and $e = uv$ such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ be any $CN-edge$. Then either u or v are in $V - S$. That is $V - S$ is common neighbourhood vertex cover of G.

Therefore, $|V - S| \geq \alpha_{cn}(G)$. Hence

$$p \geq \alpha_{cn} + \beta_{cn}....................(1)$$

Similarly; Let S be CN-vertex covering set in G and $e = uv$, such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ be any be $CN-edge$. So one of the vertices u or v most belongs to S. Then $V - S$ in G is common neighbourhood independent.

Therefore, $|V - S| \leq \beta_{cn}$. Hence

$$p \leq \alpha_{cn} + \beta_{cn}....................(2)$$

From 1 and 2 we get

$$p = \alpha_{cn} + \beta_{cn}$$

Theorem 4.21 *For any fuzzy graph G,*

$$\gamma_{cn} \leq \beta_{cn}$$

Proof. Let G be a fuzzy graph, with S is CN-independent set of V such that $|S| = \beta_{cn}(G)$. Then every vertex $v \in V - S$ is $CN-adjacent$ to at least one vertex of S.

Thus S is $CN - dominatingset$. Hance

$$\gamma_{cn} \leq \beta_{cn}$$

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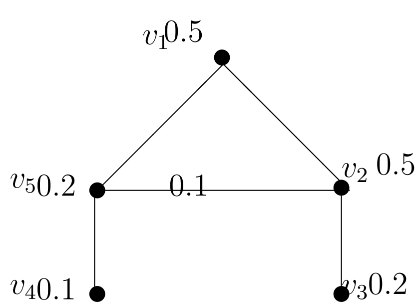
Remark 4.22 *Every CN-neighbourhood set in Fuzzy graph is CN-neighbourhood set in crisp graph*

Theorem 4.23 *Every CN-dominating set in fuzzy graph is CN-dominating set in crisp graph, but the converse is not true.*

Proof. Let $G = (\mu, \rho)$ be a fuzzy graph, with D is CN-dominating set and let $x \in D_{cn}$. Then there exists $y \in N_{cn}$ and $y \in N_{cn}(x) = \{y \in N(x); |\Gamma(x, y)| > 0\}$. Therefore, $y \in CN - neighbourhood$ set in G. By the above remark $y \in CN - neighbourhood$ set in crisp G^* so $y \in N_{cn} = \{y \in N(x)\}, |\Gamma(x, y)| \geq 1$ and x is dominates y in G^* so $x \in D_{cn}$ in G^*. Thus D_{cn} is a CN-dominating set in G^*.
In the following example, we show that the converse of the above theorem is not true.

Example 4.24 *For the fuzzy graph G given in $figure$ 4.*



$fig.$ 3.4

The vertex subset $D_{cn} = \{v_1, v_3, v_4\}$, is CN-dominating of G^*, but it is not a CN-

dominating set of G and

$D_{cn} = \{v_1, v_2, v_3, v_4, v_5\}$ *is CN-dominating set of G. .*

Theorem 4.25 *Let G be a fuzzy graph, with CN-dominating of G, then* $\gamma_{cn}(G) \leq \gamma_{cn}(G^*)$. *Furtheremore, equality holds, if* $|v| = 1$, $\forall v \in V(G)$.

Proof. Saince $\gamma(G) \leq \gamma_{cn}(G)$ and $\gamma(G^*) \leq \gamma_{cn}(G^*)$ also $\gamma(G) \leq \gamma(G^*)$. Then

$$\gamma(G) \leq \gamma(G^*) \leq \gamma_{cn}(G^*)$$

Hance

$$\gamma_{cn}(G) \leq \gamma_{cn}(G^*)$$

Theorem 4.26 *For any fuzzy graph,*

$$\gamma_{cn}(G) + \gamma_{cn}(\bar{G}) \leq 2p$$

Proof. Since $\gamma_{cn}(G) \leq p$ and $\gamma_{cn}(\bar{G}) \leq p$. Then

$$\gamma_{cn}(G) + \gamma_{cn}(\bar{G}) \leq 2p$$

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