Original Research Article

TIME SERIES MODELLING OF ACADEMIC EMPLOYEE COMMITMENT OF A SUB-SAHARAN AFRICAN UNIVERSITY

ABSTRACT

The study objective is to see how human resource management (HRM) could rely on small data evidence-based analytics to gauge employee commitment in a sub-Saharan African University. A 7-point Likert scale questionnaire on academic employee commitment in Kenya Public Universities was designed, validated and pilot tested. Out of around 60 questionnaires administered, only 31 responses were obtained before the Corona Virus (COVID-19) pandemic lockdowns in Kenya. The responses were subjected to the Modeler analyses using the statistical package for social sciences (SPSS version 21) to generate twelve optimal ARIMA (0,0,0) models for further statistical analyses. Results indicate 46.7% of employees want to spend the rest of their career in the organisation, over 61.2% of employees felt alienated and 34.9% were not emotionally attached. Around 59.3%, 64.0% and almost all employees tested on different metrics have difficulty leaving the organisation now. Although 28.9% of employees could leave abruptly, 64.6% of employees felt acculturated and 29.7% would remain at all costs. Overall, add-on effects of willingness to stay and bear with the organisation, emotional attachment, alienation, moral obligation, beneficial to remain, discouragement levels, organisational culture and being sold out to organisation could influence employee commitment levels. Thus, contributing to the HRM field, especially because the twelve-layered cascade of a series-parallel network made up of ladder and lattice structures of shared human and material resources management was used to deduce the Jackson's theorem. Future research shall consider larger sample sizes to enable us to confirm or refute the conclusions derived in this study.

Keywords: AR, ARIMA, ARMA, MA, office noise, white noise

1. INTRODUCTION

Consensus among human resource management (HRM) researchers and organisational behaviour practitioners indicate employees are the most important intellectual human capital (HC) and labour force driving the productive sectors of any economy. The success of organisations depends on HRM's ability to effectively harness HC to generate assets, goods, and services that surpass performance expectations. Employees in any industry are led and managed to enhance the achievement of set organisational goals and objectives as cutting edge[1-3].

Employees work in organisations for livelihoods[4], and commitment is employees' effort and participation in organisations[5]. Employees perceived organisational support is their overall opinions that employers appreciate their contributions and cater to their interests in terms of monetary, nonmonetary, psychological, social, work conditions, and supervisors support[6]. Further, employees' affective commitment is the emotional attachment to their organisation, while continuance commitment is employees' opinions of satisfaction with their pay structure based on costs-benefits analyses of whether to stay with their organisations. However, normative commitment tests employees' contentment with organisational values and moral obligation to stay with their organisations[7]. The University is an institution of higher learning with power to grant degrees, its body of teachers, professors, students, alumni, colleges, schools, faculties, departments, units, and other facilities[8]. Consequently, the levels of employee commitment in a University can be measured using time series analyses.

Investor capitalisation led to HRM marketisation of reward systems using highly extractive performance work systems to drive employees to the fringes of alienations and which, systematically remove them as incompetent and poor performers. The financialisation of work has driven a wedge between the cooperation that existed among owners and a combination of workers, suppliers, and supply chains. Transactional business hazards has shifted to labour because of the mistaken belief of completely replacing HC by machines, robots, or technology[9]. Consequently, employees in nonstandard employment like part-time, casual, or self-employment suffer a 30% wage penalty. Further, technology is used to control work and minds of employees to the detriment of collective bargaining, because the power of unions is short-circuited and technology is not known to grow talent nor support collaborative HC development.

Reward management relates perceptions of distributive justice of fair employees remunerations package, while procedural justice suggests assets allotment, how they influence and affect social relationships (interactional justice) and employee contentment and commitment. The foregoing has subjective and objective components (performance and market rates) based on culture and belief systems of value creation and wealth distribution[9]. Empirical evidence indicates that even if organisations offer the most expensive and attractive rewards, employees may not be adequately engaged if not well communicated. Most commonly operated rewards trends emphasise employees understand and support how their pay and rewards were determined. Thus, 70% of HR professionals indicate few employees understand their reward policies and strategies, 56% opine reward communications were ineffective, 43% disagree employees understood how their pay was linked to performance, while 44% employers were making deliberate changes to their reward communication systems[10].

Two major career paths to dream role are either a vertical ladder of climbing ranks through a department or contemporary lattice platform in which valuable skills, and experiences are gained through different positions and separate departments. Although the ladder route seems excellent in climbing corporate structure relatively rapidly, the employee is limited for not learning new skills or experiencing new exposures in other disciplines than they were trained. Knowledge of the organisation is narrow and less valuable to employers. Conversely, lattice can be sideways, horizontal, backward, and forward movements in different departments, separate roles, contrasting experiences, and diverse exposures[11].

Employees with lattice careers are more valuable to the organisation, not easily bored, more robust, and rounded in their knowledge of the organisation. They use several perspectives to view problems confronting organisations with more balanced and pragmatic approaches at offering solutions based on corporate strategy. Export of talent and expertise to other organisations is less likely and organisation benefits immensely from investment in employee development and growth with their organisations. Lattice careers are the future because flatter organisations mean horizontal growths are preferred, employees are expected to be well rounded with a bundle of skills applicable in many different areas of the enterprise[11]. A hybrid approach could be used by the employee to climb the ropes in the face of job security and lean organisations to reduce costs for the multitude of work to be done by fewer people in the shortest possible time and highest levels of professionalism.

Time series is a sequence of measurements recorded over time for predicting future values of socio-economic importance, which are used for cost savings or quality products and services offerings critical to organisational success[12]. They are sets of time gaps analysed for more effective business decision optimisation, and are categorised into time (autocorrelation and cross-correlation analyses) and frequency (spectral and wavelet analyses) domains[13]. Time series can model future behaviour containing autocorrelation, trends, cyclical, seasonal, and stochastic components[14].

They provide solutions to difference equations employing autoregressive (AR), moving average (MA), Box-Jenkins, autoregressive and moving average (ARMA), and autoregressive-integrated (differenced) - moving average (ARIMA) constructs to predict future values[12]. Autoregression model is variable regression against itself in which forecast variables use linear combinations of past values of independent uncorrelated predictors while MA uses past forecast errors to predict future values. Each value of output is viewed as a weighted MA of past few forecast errors[15].

Further, ARMA processes combine AR and MA models. Integration (adding) or differencing (subtraction) of AR and MA components form the ARIMA model, which uses only information in past time sequence to predict future values. They reduce non-stationary series using a sequence of differences and first difference of non-stationary Gaussian random walk to stationary white noise[16] to identify and choose optimal models by selecting parameters of Box-Cox transform (integration or difference) that pinpoint desired stationary ARMA processes. Since data spread does not fluctuate around the time axis[17], ARIMA is synthesised using coordinated time sequence, stationarity, model identification, parameter estimation, and diagnostic checks[18]. The independent variables are stationarised while independent variables are error lags. If the lag is zero, we cannot forecast one month ahead[19].

The parameter model tests statistical significance, residuals indicate white noise employing Box-Jenkins Q metric and the lowest Bayesian information criterion (BIC) is the most optimal model. Ljung-Box Q tests for lack of fit in models and assesses residuals autocorrelation, in which very small values do not indicate a significant lack of fit[20]. ARIMA (p,d,q) models adjust observed values to reduce the difference between observed and generated model values. The series is stationary if mean and variance do not change with time, and their covariance depends on the gap between two periods[21].

ARIMA models compare mean error (ME), mean absolute error (MAE), or square mean error (SME). It is flexible in monitoring and observing data patterns, which offer valuable insights for future planning and awareness[22]. ARIMA model determines the order of differencing to obtain stationary sequence, which causes oscillation about the mean, while autocorrelation function (ACF) falls rapidly to zero, either from top or bottom. If lag-1 autocorrelation is zero or negative, the sequence needs no further differencing and the optimal order of differencing is the lowest standard deviation[19].

Standard error spreads values within a set of data and estimates population parameters interval locations and precision of sample statistic representativeness in a population[23]. R-square measures the variables explanatory power within the investigated population, while differences from unity are unexplained model variables. Stationary R-square compares stationary parts of the model to a simple mean model, while ordinary R-square measure is not due to stationary data[24].

Root mean square error (RMSE) evaluates how the dependent sequence varies from the model prediction level. Mean absolute percentage error (MAPE) tests how the dependent sequence differs from the predicted model and measures the quality of goodness-of-fit in dimensionless quantity. MAE suggests a percentage by which a series differs from the predicted model. The maximum absolute percentage error (MaxAPE) is the largest forecast error in percentages. The ARIMA model worst-case error usually derives from maximum absolute error (MaxAE). The normalised Bayesian information criterion (Normalised BIC) evaluates the overall model fit that accounts for polynomials complexity, penalty factor and length of the sequence[25-30].

The stochastic signals process estimation in the presence of noise is a critical problem, and noise can be coloured or white. Coloured noise is correlated with the input, while white noise is uncorrelated and independent of the input. Analyses of white noise lead to filters like matched filters, Kalman filters, ladder filters, lattice filters, and whitening filters[31-32]. White noise is the random signal of equal intensity occurring at different frequencies as flat or constant power spectral density. White noise series is stationary because it appears the same irrespective of when it was observed and has no predictable patterns in the longer terms. Time plots are unevenly horizontal while some cyclic behaviour is possible because of constant variance[15].

Whenever Ljung-Box tests have significantly large p-values, there is strong evidence of discrete white noise and the best ARIMA model has zero differencing[16]. ARIMA analyses predict equivalently spaced univariate time sequence that supports seasonal, subset, interrupted sequence models, and multiple ARMA regression analyses of whichever complexity.

White noise tests whether ACFs are close to zero up to some lag. If this condition is satisfied for all lags, no further tests or information is generated and statistical software package stops automatically without statistics allowed for future predictions. Furthermore, ARIMA handles time sequences of moderate sizes at least 30 observations while less than 30 observations, parameter estimates are poor and thousands of observations lead to large computer simulation time, huge memory requirements, and high capital expense[31].

Presently, HR departments use technology to provide operational efficiencies and optimise processes like controlling noise, which are discordant sounds of variable frequencies. Direct-field sound mask protects speech privacy, reduces office noise distraction, leads to comfortable and employee-friendly work environment for productivity[33]. Conversations can occur between employees because management has introduced new strategies, mergers, acquisitions, innovations, or other changes. If they are not talking among themselves, they are thinking and may be unable to concentrate on their tasks, leading to poor productivity and performance levels. However, conversation noise in organisations could be harnessed to build trust to guide organisations new purpose and vision[34].

Workplace noise levels are health and safety problems. They occur from universal increase and the strong influence of low-level noise from substances present in the work environment. Some global studies that included Australia indicate 20% of employees comprising senior executives and lower rungs across industries in 2015 could focus on tasks regardless of office noise. In 2017, that figure significantly reduced to 1%[35]. Further, 63% of employees indicate open-plan offices deny them quiet and highly intensive work workplaces thereby negatively impacting creativity, contentment, and welfare. Even sharp noises from spiky high-heeled shoes became a nuisance. The researchers also discovered that 75% of employees either go for a walk or to a coffee shop to calm their nerves, 32% used headphones to damp out distractions, and HR is in a quandary because many employees operating in the noisiest office environments were willing to quit their jobs within the next six months[35].

The objective of the study is to see how human resource management could rely on small data (pilot) to gauge academic employee commitment of a sub-Saharan African University. Time sequence models are substitute management evaluation programmes for examining previously observed or unobserved variables that provide dependable results. This characteristic enables investigators to identify, explain, and forecast the consequences of management programmes over time. Use ARIMA constructs to substantiate optimal solutions to HRM predictive analytics problems, while deploying statistically computed error metrics to evaluate goodness-of-fit of the cascaded ARIMA models. Overall, the add-on effects of willingness to stay, willingness to bear, emotional attachment, alienation, moral obligation, beneficial to remain, discouragement levels, organisational culture, and being sold out to the organisation could influence academic employee commitment levels. The study is organised into Introduction, Materials and Methods, Results and Discussion, and Conclusion.

2. MATERIALS AND METHODS

A seven-point Likert scale questionnaire on academic staff employee commitment in Public Universities was designed, validated and pilot tested in a Public University within Nairobi, Kenya. The 7-point Likert scale is defined as follows: 1 = Strongly disagree; 2 = Disagree; 3 = Disagree somewhat; 4 = Undecided; 5 = Agree somewhat; 6 = Agree; 7 = Strongly agree.

The questionnaire was designed to measure the research objective because it is a reliable instrument for gathering the opinions and perceptions of respondents regarding the subject under investigation. It is also a means of obtaining qualitative data, which can be transformed into quantitative data using the Likert scale, so that it can be amenable to statistical analyses.

The sample size for the main study is 358 and 10% sample size was used for the pilot study (36). About 60 questionnaires were randomly administered and only 31 of the returned questionnaires before the Corona Virus pandemic (COVID-19) lockdowns were used for statistical analyses. The Modeler in the statistical package for social sciences (SPSS version 21) was used to model input data to generate ARIMA model results and plots for the study.

2.1. SAMPLE SIZE ADEQUACY

Uncertainties are unavoidable in measurements because they influence the conclusions we draw from data with finite sample sizes. They result in average and root mean square deviation (RMSD):

$$\sigma = \left\{\frac{1}{N}\sum_{i=1}^{N}(l_i - \mu)^2\right\}^{\frac{1}{2}} \tag{1}$$

where μ is sample average, l_i average results, and N sample size[36]. But, how likely are average value measurements true estimates? Without additional information, we reject sample average μ as a mean of the parent distribution. The Normal distribution is invoked if errors are many and independent[14]. If sampled data is random amidst parent distribution members, the spread of mean (μ) and variance (σ), are excellent estimators[36].

For the normal distribution, probability range is $[\mu - \sigma, \mu + \sigma]$:

$$A(\sigma) = \int_{\mu-\sigma}^{\mu+\sigma} P_G(\mu, \sigma, t)dt = 0.68 \tag{2}$$

where P_G is probability function and t is time. If measurements are conducted 100 times, each consisting of 31 measurements, 68% lie between $(\mu - \sigma)$ and $(\mu + \sigma)$. For only one possible sampling set from the parent distribution, we assume Poisson distribution[36].

Let $\mu \equiv Np$ (3)

As probability: $p \to 0$, $N \to \infty$, μ is constant and binomial distribution approaches Poisson distribution:

$$P_p(n, \mu) = \frac{\mu^n}{n!}e^{-n} \tag{4}$$

Measurements distribution is Poisson for infinite samples of 31 surveys of employee commitment. Further, the mean and variance of Poisson processes are equal. Therefore, distribution is completely determined by one value and it approaches normal distribution when degrees of freedom (DF), is large. Consequently, its occupancy for any sample is $n^{1/2}$ and 68% probability that the true value is within $\left[31 - (31)^{\frac{1}{2}}, 31 + (31)^{\frac{1}{2}}\right]$[36]. Since the true sample size lies between 25 and 37, the 31 samples used for the study are adequate to draw pertinent conclusions.

2.2. AUTOREGRESSIVE (AR) MODEL

Autoregressive (AR) model of order p, AR (p) is

$$X_t = c + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \cdots + \varphi_p X_{t-p} + \epsilon_t; t = 1,2, \ldots, N \quad (5)$$

where c is constant, φ_i coefficients, ϵ_t is the error and X_t are predictors. If ϵ_t is white noise (WN) series, independent and identically distributed (iid) random variables, expectation value $E\{\epsilon_t\} = 0$, and variance $(\epsilon_t) = \sigma^2$. Thus, $\epsilon_t \sim iid\ WN(0, \sigma^2)$. Also, long-term memory models are additive on X_t by all previous values[15].

2.3. MOVING AVERAGE (MA) MODEL

Time sequence $\{X_t\}$ of moving average (MA) process of order q, is

$$X_t = \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \cdots - \theta_q \epsilon_{t-q} \quad (6)$$

where θ_i are coefficients and model is described by past errors as explanatory variables. It is a short memory model because only q errors affect X_t as higher-order errors do not[37].

2.4. AUTOREGRESSIVE AND MOVING AVERAGE (ARMA) MODEL

An ARMA (p, q) model is a time sequence $\{X_t\}$, if

$$X_t = c + \varphi_1 X_{t-1} + \cdots + \varphi_p X_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q} \quad (7)$$

Equation (7) is a combination of AR and MA models[32].

2.5. AUTOREGRESSIVE INTEGRATED MOVING AVERAGE (ARIMA) MODEL

We combine differencing with autoregression and moving average models to obtain the non-seasonal ARIMA model (integration is reverse of differencing). Thus,

$$X_t' = c + \varphi_1 X_{t-1}' + \cdots + \varphi_p X_{t-p}' + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} + \epsilon_t \quad (8)$$

where X_t' is differenced sequence. Lagged predictors and errors are on the right of equation (8) of ARIMA (p, d, q) model. Also, d is differencing that affects prediction intervals which are almost the same if d is zero[13].

3. RESULTS AND DISCUSSION

This section contains the analyses of results and discussion of Tables I-III and Figure 1.

Table I describes twelve models, Model_1 to Model_12 and each of $ARIMA(0,0,0)$ types. They are all stationary white noise time series because p, d, q are all equal to zero, and no need to difference the time sequence. Consequently, long-term effects go to zero (if c is zero and d is zero). Further, longer terms forecast standard deviation goes to the standard deviation of historical data while prediction intervals are almost the same[15]. However, the best ARIMA model has zero differencing[16]. Models without an order of differencing assume original sequence is stationary (mean-reverting) and each model comprises constant term to account for non-zero mean value. The estimated regression equation for each of the twelve $ARIMA(0,0,0)$ models was (If d is zero); $\hat{Y}_t$ equals $\hat{y}_t$ or y_t equals ϵ_t. Zero-order difference is original series and $ARIMA(0,0,0)$ is without differencing, no AR or MA terms, only a constant. It is the mean model and time sequence plots of residuals from the mean[19].

Applying it to seasonal random walk $ARIMA(0,0,0) \times (0,1,0)$, a seasonal difference for monthly data at period t is Y_t minus Y_{t-12}, so that $\hat{Y}_t$ minus Y_{t-12} equal μ. It assumes each season is a random step away from saying March last year while April value this year is also a random walk away from April value last year and mean of every step equals μ. Therefore, all forecasts for March 2020 overlook all data after March 2019 because it is based entirely on what happened one year ago. The model does not respond rapidly to cyclical variations in data. It was identical one year behind, assuming current trends equal regular trends and predictions could err in the same direction for several months in a row. Consequently, longer-term predictions beyond the end of the sample are more plausible as they expect regular trends in the past will ultimately occur again in the future. However, the seasonal random walk white noise ARIMA model is relatively stable in the presence of sudden shocks in data because it does not notice them for twelve months. This result also corroborates the same Poisson input and the same Poisson output $M/M/r$ queuing model used in the network cascade (Figure 1) to realise the Jackson's theorem[32] in the study.

The AR approach was used in the analyses because ARIMA(0,0,0) is stationary white noise without trends. Stationarity indicates that the series is stationary, and the series is normally distributed, and the mean and variance are constant over a long time period. There are no autocorrelations in the series, and therefore, a linear process. Furthermore, autoregression provides an avenue of examining a broad class of linear functions, and used to select the best linear method that works best from the class of forecasts for the current sample[15-19].

Table II displays the Model Fit statistics for the twelve-tier ARIMA$(0,0,0)$ models. The statistics consist of stationary R-squared, R-squared, RMSE, MAPE, MaxAPE, MAE, MaxAE, Normalised BIC and SE, and Mean. Stationary R-square compares the stationary part of a model to a simple mean model, while ordinary R-square is not due to stationary data. R-square estimates the proportion of total variation in sequence as explained by the model is most valuable after the sequence is stationarised. If stationary R-square is greater than R-square, then the employee commitment under investigation is better than the baseline model[24,29].

Stationary R-square indicates between very infinitesimally small quantity greater than zero percent and 64.6% variation of cascaded ARIMA models were accounted for by stationary data. Between less than one hundred percent and 35.6% variation were due to unexplained variables.

Ordinary R-square suggests between slightly less than zero percent and 64.6% variation of the cascaded ARIMA models was accounted for by time sequence data. Between one hundred percent and 35.6% variation were from other estimators. Further, mean values for stationary R-square and ordinary R-square are 39.8% and 39.2%, respectively.

Although the range between stationary R-square and R-square values was almost the same, the means were different. Once stationary R-square is greater than ordinary R-square, the ARIMA $(0,0,0)$ models under investigation are better than their baseline models[24,29]. However, negative R-square is possible for regressions without intercept in the model or constant. It suggests the model fits data rather poorly and model regression line measure is worse than using the mean value as a metric[38]. Small and negative R-square values indicate a slight or insignificant proportion of total variation was allowed by the model. Also, models under investigation are worse than baseline models[24].

Root mean square error (RMSE) evaluates how the dependent sequence varies from its model predicted level. The original employee commitment time series data differed from their predicted model levels by between 1.0 and 1.9%. This is a considerably very good result assuming a smooth sequence[19,29].

Mean absolute percentage error (MAPE) examines how the dependent sequence differs from its predicted model level. It is an unbiased estimator of model goodness-of-fit because it is dimensionless. Consequently, MAPE of study ranged between 19.8 and 79.6%, while the average of MAPE was around 53.9%. Therefore, MAPE obtained in the study is satisfactory because there is no upper limit[27].

MaxAPE for the study ranges between 70.4 and 408.1%. They are the largest forecast errors in percentages and worst-case projections. Mean absolute error (MAE) suggests series differs from the model predicted level by between 0.8 and 1.7%, while the mean was about 1.2%. Since RMSE was at least as big as MAE in the study, it suggests model forecasts are satisfactory and consistent. They are only equal if all errors are the same[26].

Maximum absolute error (MaxAE) is worst case-time series prediction for study and ranges between 2.1 and 4.1%, while mean was about 3.3%. If MaxAE is greater than MaxAPE for small values, then MaxAE appears at large sequence values and *vice versa*[27]. In the study, MaxAPE values were greater than all MaxAE values by factors of approximately between 2.5 and 2.8. We can infer that the time sequence data were fairly uniform, stable, or differed by an order of magnitude.

Normalised BIC determines parameterised model forecast data considering number of model parameters. It is used to select the overall optimised fit models based on mean square error, parameter number minimisation, and length of the sequence, and ranged between 0.185 and 1.478 in the study.

Table III displays the twelve optimal ARIMA models and their respective statistics.

Box-Ljung determines independence of all lags up to that specified, measures complete randomness based on the number of lags up to 20, and assesses whether ARIMA residuals have autocorrelations.

P-values above .05 for Ljung-Box-Pierce tests suggest non-significance and are considered good results without patterns in residuals so that predictions can be made. Small P-values less than .05 suggest nonzero autocorrelation within the first m lags[13,39]. If Q is greater than $\chi^2_{DF,\alpha}$ at specified degrees of freedom (DF), and significance level (α), the null hypothesis is rejected. When P-values are greater than .05, the Ljung-Box coefficients have little influence on model description (overfitting)[15,40]. Further, chi-square from table at 18 DF and $\alpha = 0.05$ is: $(\chi^2_{DF=18,\alpha=0.05} = 28.869)$.

Therefore, Model_1 indicates 46.7% was contributed by stationary data, while 53.3% was due to other unexplained factors. Q-statistic was less than chi-square and probability was insignificant. Thus, model_1 has no autocorrelations lying outside the 95% limits[15] and Ljung-Box (Q) statistic P-value (.983), suggest employees willingness to spend rest of their career with University is random and may not correlate with those of previous years.

Model_2 indicates 57.1% was accounted for by stationary data, while 42.9% are unexplained components. Q-statistic was below chi-square value and probability was insignificant[39]. Since no autocorrelations lie outside 95% limits because the Ljung-Box statistic P-value (.606) implies employees' willingness to bear with University is by chance and could not be correlated with those of earlier years.

Model_3 explains 61.2% was contributed by stationary data, while 38.8% were from unknown parameters. There are no autocorrelations beyond the 95% limits, the P-value was .804, and Q statistic was lower than chi-square value $(DF = 18)$, leading to optimal model[17]. Therefore, employees' alienation at University is random and may not correlate with past years.

Model_4 describes 34.9% of stationary data while 65.1% are from other factors. Q statistic is below chi-square value, P-value (.608) indicates non-significance without autocorrelations outside the 95% limits presupposes satisfactory ARIMA model[20]. Also, non-emotional employee attachment to University suggests a random process that may not correlate with earlier years.

Model_5 explains 59.3% of stationary data, while 40.7% were unexplained factors. Chi-square is higher than the Q statistic, while P-value (.281) implies rejecting the null hypothesis, and a good Ljung-Box model fit to data[41]. Levels of employee involvement with the organisation would make spot decisions not able to correlate with former years.

Model_6 reflects 64.0% total variation was from stationary data, while 36% were from unknown parameters. Chi-square was greater than Q value and P-value (.144) indicates insignificance of the null hypothesis, guarantees optimal model fit for the sequence[26]. Employees were unable to quit their jobs now because of the costs and chances of previous years' occurrences could not be correlated.

Model_7 reflects an infinitesimally very small (1.0E-011%) percentage of stationary data, while close to 100% total variation was due to external factors that may not be white noise, but coloured[42]. Chi-square was higher than Q-value and P-value (.290) rejects the null hypothesis as insignificant for optimal model fit to data[31]. Slight employee costs make quitting jobs much easier than previous years and the chances are uncorrelated.

Model_8 shows 28.9% of stationary data, while 71% were due to unknown predictors. Chi-square was still higher than Q-value and P-value (.850) rejects the null hypothesis as insignificant, while the model fit was deemed optimal[40]. Employee alienation and frustration that may heighten turnover never previously experienced were uncorrelated.

Model_9 suggests a 30.9% variation was contributed by stationary data while 69.1% were due to unexplained factors. Chi-square was below Q-value, while P-value (.021) was significant and we accept the null hypothesis, but Ljung-Box (Q) shows a significant lack of fit to data and rejects randomness hypothesis. That means autocorrelations for some lags may be significantly different from zero and the values are not random and independent over time. Also, the observation can be correlated with separate observations m time units, later (autocorrelations). Further, autocorrelation decreases the precision of time-based predictive models and misinterpretation of data[43]. Therefore, employee loyalty and moral obligation to organisation are not random and could be correlated with future observations.

Model_10 indicates a 64.6% contribution derives from stationary data while 35.4% variation was due to unexplained estimators. Q-value was greater than chi-square while P-value (.000) was significant, but suggests a significant lack of fit, non-zero autocorrelations within the first 18 lags and possible misinterpretation of results[39]. Therefore, employee cultural values of organisation are not random and independent because present cultural experiences could correlate with future experiences.

Model_11 indicates 29.7% was stationary data while 70.3% variation was due to other predictors. Q-value was significantly higher than chi-square and P-value (.000), was significant and reflects a considerable lack of fit with autocorrelation effects[20]. Thus, employee beliefs in the organisation are not random and independent because current organisational beliefs could correlate future beliefs.

Model_12 describes a near-zero infinitesimally very small percentage (1.0E-011%) stationary data while very close to 100% variation was due to unknown estimators. Q-value was higher than chi-square while P-value (.000) indicates a significant model lack of fit with autocorrelation effects[41]. Consequently, employees who sold out to the organisation scarcely exist, as occurrences are not random and independent because prevailing observations could correlate future patterns.

Figure 1 indicates a cascade of twelve ARIMA$(0,0,0)$ stationary time series properties that do not depend on the time the series were observed. This is so because a white noise series looks approximately the same irrespective of when it is observed. Further, each of the series in Figure 1 has no predictable pattern in the longer terms. The time plots show that the series are unevenly horizontal and the timing cycles are unpredictable because the series is stationary[15].

3.2. PROMOTIOL POLICY

The promotional policy in the University is dependent on the academic employee's attainments that satisfy certain laid down criteria. The employee applies first for promotion and then an internal advertisement is displayed internally. It is also to notify the Human Resource Directorate, which makes announcement through internal advertisement before the interview that comes after six months of the announcements.

But, annual increments are automatic because there are no applications for them. For promotion, advertisement must be made for someone to progress from one level to a higher level.

Table 1: Model Description

Model ID		Model Type
I would be very happy to spend the rest of my career with this university	Model_1	ARIMA(0,0,0)
I really feel as if this university's problems are mine	Model_2	ARIMA(0,0,0)
I do not feel like "part of the family" at this university	Model_3	ARIMA(0,0,0)
I do not feel "emotionally attached" to this organisation	Model_4	ARIMA(0,0,0)
It would be very hard for me to leave my organisation right now, even if i wanted to	Model_5	ARIMA(0,0,0)
Too much in my life would be disrupted if I decided to leave this university now	Model_6	ARIMA(0,0,0)
It would not be too costly for me to leave this university now	Model_7	ARIMA(0,0,0)
I am not afraid of what might happen if I quit my job without having another one lined up	Model_8	ARIMA(0,0,0)
One major reason i continue to work in this organisation is my belief that loyalty is important and moral obligation	Model_9	ARIMA(0,0,0)
I was taught to believe in the value of this organisation	Model_10	ARIMA(0,0,0)
I do not feel it is right to leave my institution if I get a better job offer elsewhere	Model_11	ARIMA(0,0,0)
I do not think to be an organisation man /woman is sensible anymore	Model_12	ARIMA(0,0,0)

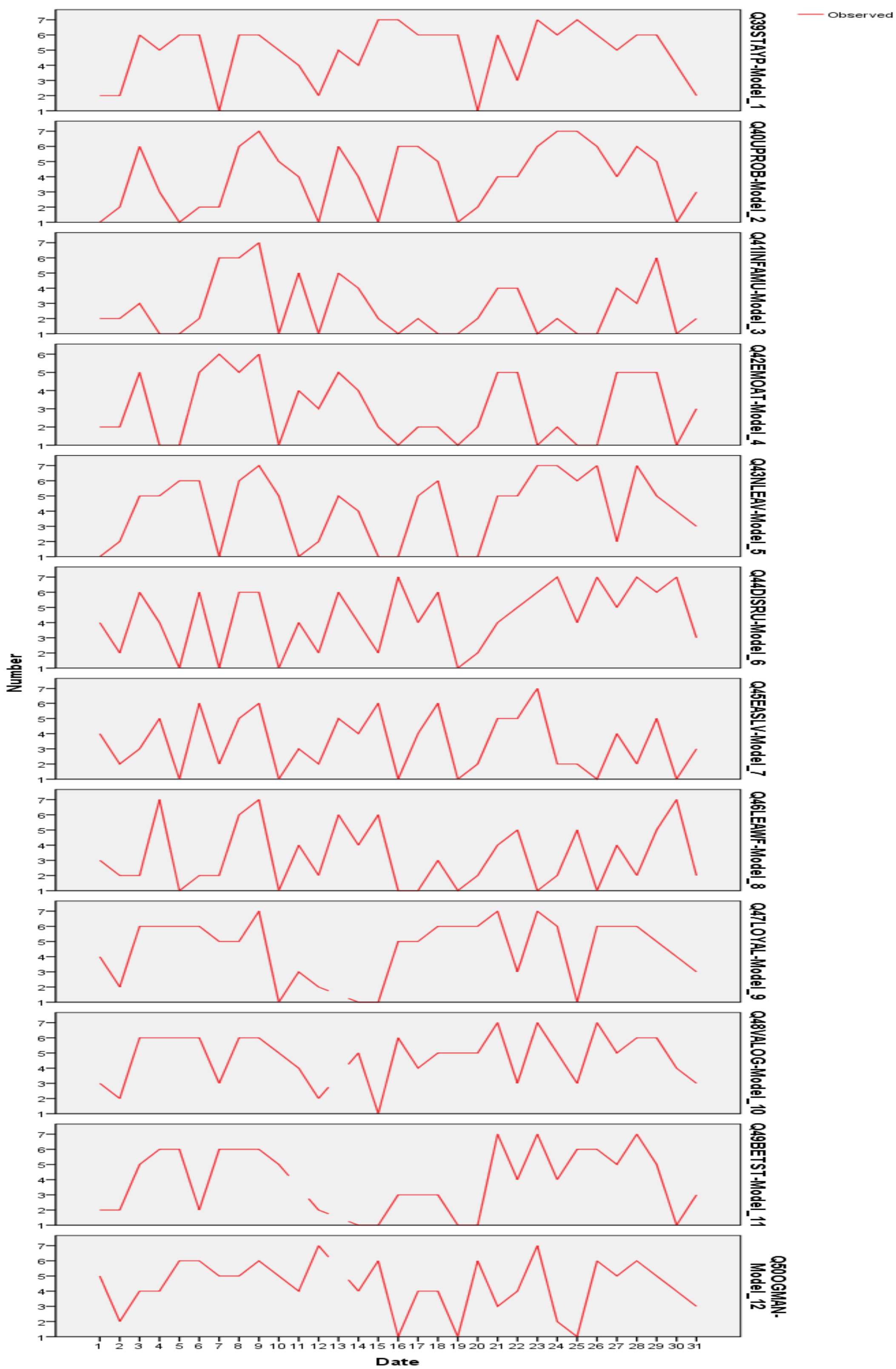
Table 2: Model Fit

Fit Statistic	Mean	SE	Minimum	Maximum	Percentile						
					5	10	25	50	75	90	95
Stationary R^2	.398	.231	1.006E-013	.646	1.006E-013	1.012E-013	.291	.408	.607	.644	.646
R2	.392	.234	-2.498E-005	.646	-2.498E-005	-1.748E-005	.244	.408	.606	.644	.646
RMSE	1.529	.272	.979	1.876	.979	1.063	1.317	1.535	1.777	1.865	1.876
MAPE	53.889	17.736	19.816	79.618	19.816	24.446	42.181	52.832	68.860	78.248	79.618
MaxAPE	293.987	92.407	70.431	408.101	70.431	114.669	249.155	304.714	361.784	406.540	408.101
MAE	1.193	.256	.764	1.659	.764	.811	1.012	1.167	1.416	1.596	1.659
MaxAE	3.265	.571	2.113	4.081	2.113	2.276	2.816	3.261	3.669	4.065	4.081
Normalized BIC	1.112	.371	.185	1.478	.185	.334	.923	1.215	1.353	1.468	1.478

Table 3: Model Statistics

Model	Number of Predictors	Model Fit statistics	Ljung-Box Q(18)			Number of Outliers
		Stationary R-squared	Statistics	DF	Sig.	
I would be very happy to spend the rest of my career with this university-model_1	1	.467	7.671	18	.983	0
I really feel as if this university's problems are mine-model_2	2	.571	15.808	18	.606	0
I do not feel like "part of the family" at this university-model_3	2	.612	12.793	18	.804	0

I do not feel "emotionally attached" to this organisation-model_4	1	.349	15.779	18	.608	0
It would be very hard for me to leave my organisation right now, even if I wanted to-model_5	2	.593	20.976	18	.281	0
Too much in my life would be disrupted if I decided to leave this university now-model_6	2	.640	24.336	18	.144	0
It would not be too costly for me to leave this university now-model_7	0	1.027E-013	20.794	18	.290	0
I am not afraid of what might happen if I quit my job without having another one lined up-model_8	1	.289	11.952	18	.850	0
One major reason I continue to work in this organisation is my belief that loyalty is important and a moral obligation -model_9	2	.306	32.230	18	.021	0
I was taught to believe in the value of this organisation-model_10	2	.646	129.191	18	.000	0
I do not feel it is right to leave my institution if I get a better job offer elsewhere-model_11	2	.297	124.650	18	.000	0
I do not think to be an organisation man /woman is sensible anymore-model_12	0	1.006E-013	62.783	18	.000	0



Observed
Q39STAYP-Model_1
Q40UPROB-Model_2
Q41INFAMU-Model_3
Q42EMOAT-Model_4
Q43NLEAV-Model_5
Q44DISRU-Model_6
Q45EASLV-Model_7
Q46LEAWF-Model_8
Q47LOYAL-Model_9
Q48VALOG-Model_10
Q49BETST-Model_11
Q50OGMAN-Model_12
Number
Date

Figure 1. Times series modelling cascade of academic employee commitment of a sub-Saharan African University

4. CONCLUSION

The results show $ARIMA$ (0,0,0) models were stable and optimal solutions to the employee commitment problem under investigation because they do not respond rapidly to cyclical variations in data. Longer-term predictions beyond the end of the sample are more plausible because random walk white noise ARIMA model does not respond to sudden shocks in data since it does not notice them for twelve months.

The above confirms our claims of using the generated network cascade to deduce Jackson's theorem of the same Poisson input and the same Poisson output $M/M/r$ queuing model in the study. While 46.7% of employees were willing to spend the rest of career in organisation, 57.1% felt obliged to carry organisational burden as theirs. Over 61.2% of employees felt alienated and 34.9% were not emotionally attached. Around 59.3% of employees find it difficult to leave organisation now, and 64.0% of employees indicate a great disruption in their lives if they left the organisation.

Almost every employee felt it would be very costly to leave organisation now, but 28.9% of employees were prepared to leave without alternative employments in view. Over 30.6% of employees felt loyalty was a moral obligation while 64.6% of employees felt obliged to imbibe the values and culture of organisation. Also, 29.7% of employees were prepared to remain even if they got jobs elsewhere, while virtually no employees were ready to be completely sold out to the organisation.

The twelve-layered cascade is a series-parallel network made up of ladder and lattice structures of shared human and material resources. We recommend lattice structure or a hybrid to reduce talent and expertise export, benefit from investment in employee development and growth by maximising employees well-rounded bundle of skills applicable in many different areas of lean organisations to reduce costs and retain highest levels of professionalism to boost employee commitment in the organisation. Future research shall consider a much larger sample size study to either corroborate or refute findings by reducing bias, data sources not singular, or restricted to a psychological view of HRM.

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APPENDIX

APPENDIX A

A.1. STANDARD ERROR (SE)

SE estimates the interval of population parameter locations and representativeness. The standard deviation assesses the variability of the sample. The standard error of the mean (SEM):

$$SE = \frac{\sigma}{\sqrt{n}} \quad \text{(A.1)}$$

where σ is a standard deviation, n is the sample size, σ_M is the standard error of the mean (SEM). We use ($SEM \pm 1.96$) to determine 95% sample population mean lies between upper and lower limits. Small standard errors provide accuracy of statistics and precision of significance of findings[23].

A.2. ROOT MEAN SQUARED ERROR (RMSE)

RMSE measures how far the model is from its next prediction and is the standard deviation of residuals or prediction errors. It evaluates data spread of residuals around a line of best fit:

$$RMSE = \sqrt{\frac{1}{F}\sum_{i=1}^{F}(\epsilon_t)^2} \quad \text{(A.2)}$$

where F is out-of-sample observations reserved for evaluating prediction step[26].

A.3. COEFFICIENT OF DETERMINATION (R-SQUARED)

R-squared tests how far the optimal model regression line is from a simple horizontal line drawn through mean data and values greater than zero indicate regression analyses are better metrics. R-square suggests how much error was subtracted from unity using regression analyses,

$$R^2 = 1 - \frac{SS_{Regression}}{SS_{Total}} \quad \text{(A.3a)}$$

Total sum squared error (SS_{Total}) is subtracting mean value from each point and squaring their results,

$$SS_{Total} = \sum(y_i - \bar{y})^2 \quad \text{(A.3b)}$$

where y_i is each data point, $\bar{y}$ is mean value, $\sum$ is a sum over all data points and a square of the difference between each data point and mean value. Sum squared regression ($SS_{Regression}$) error equation equals equation (A.3b), except using a regression for forecasting instead of the mean value.

$$SS_{Regression} = \sum(y_i - y_{regression}) \quad \text{(A.3c)}$$

where $y_{Regression}$ is regression value[38].

A.4. MEAN ABSOLUTE PERCENTAGE ERROR (MAPE)

Mean absolute percentage error (MAPE) is dependent sequence variation from its predicted model level. It uses dimensionless quantities for measurements.

$$MAPE = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{A_i - F_i}{A_i}\right| \quad (A.4)$$

where A_i is actual value and F_i is a forecast value. MAPE is zero for a perfect fit and has no upper limit[27].

A.5. MEAN ABSOLUTE ERROR (MAE)

MAE is sequence variation from the predicted model level in original sequence units and evaluates variation of predictions from eventual outcomes.

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|F_i - A_i| = \frac{1}{n}\sum_{i=1}^{n}|\varepsilon_i| \quad (A.5)$$

where ε_i is absolute error, F_i is prediction value and A_i is computed value. It forecasts errors in time sequences[27].

A.6. MAXIMUM ABSOLUTE PERCENTAGE ERROR (MaxAPE)

MaxAPE is the largest forecast error in percentages, perceived as worst-case forecasts.

$$MaxAPE = max\left(\left(\frac{|y_i - \widehat{y_i}|}{y_i}\right) \times 100\right) \quad (A.6)$$

where y_i is predictor and $\widehat{y_i}$ is an estimated value of predictor[44].

A.7. MAXIMUM ABSOLUTE ERROR (MaxAE)

MaxAE is the largest forecast error described by the same units as the dependent sequence. It visualises the worst forecast cases.

$$MaxAE = \max_{1 \le i \le N}\{|\widehat{d_i} - d_i|\} \quad (A.7)$$

where d_i is sequence data and $\widehat{d_i}$ is reconstructed value for d_i[45].

Both $MaxAE$ and $MaxAPE$ could appear at separate points in a sequence. If $MaxAE > MaxAPE$ for small values, then $MaxAE$ appear at larger sequence values and *vice versa*[27].

A.8. NORMALISED BAYESIAN INFORMATION CRITERION (Normalised BIC)

Normalised BIC is an overall model fit test for polynomial complexity that depends on MSE, parameter numbers penalty, and length of the sequence.

$BIC = \chi^2 + kln(n)$ (A.8a)

where χ^2 is chi-square distribution, k is intercept, n is observations. BIC selects the optimal model if k is constant. It parameterises model efficiency of forecast data and the number of model parameters[27]. Normalised BIC becomes:

$Normalised\ BIC = \frac{BIC}{\sqrt{n}}$ (A.8b)

where $\sqrt{n}$ is normalisation factor[46].

A.9. Ljung-Box$(\mathbf{Q})$ TEST

The Ljung-Box metric is used when P-values are above 0.05, strong evidence of discrete white noise, and a good fit to residuals[16]. Time series plots of residuals indicate no trends in residuals, no outliers, and no change in variance. Ljung-Box-Pierce statistics are for lags up to 20[39], and are accumulated sample autocorrelation functions (ACFs) of r_j up to any time lag m:

$Q(m) = n(n+2)\sum_{j=1}^{m}\frac{r_j^2}{n-j} \cong \chi_n^2$ (A.9)

where n is usable data after differencing. Null hypothesis distribution $Q(m)$ approximates chi-square (χ^2) distribution with DF $(df = m - P)$ and P is coefficients in the model. The statistic is undefined unless $m > P$. Without using any model, ACF is raw data $P = 0$, and null hypothesis $Q(m)$ approximates χ^2 distribution with $df = m$[26].

A.10. CASCADED NETWORKS

The Poisson process is a stochastic point process if a queuing event has just occurred or that it may not have occurred for a long time, does not guarantee that it will not occur soon. The queue symbol M indicates Poisson or exponential (Markovian or memoryless) term and $M/M/r$ is Poisson arrivals, Poisson exits, and r number of servers in a waiting queue. For $M/M/r$ queues all input and output processes are Poisson while equivalent characteristic λ is in a stable state[32]. Network connection cascade (Figure 1) has $(m = 12)$ tiers of shared resources by a set of customers or employees, which serve several parallel operating points. If the input to the first deck is Poisson, all intermediate inputs and outputs to consequent tiers are Poisson of equivalent rate and each tier acts like $M/M/1$ queue in a stable state. Expected components of m –tier series interconnection, is

$L = \sum_{n_1,n_2,\ldots,n_m}(n_1 + n_2 + \cdots + n_m)p(n_1, n_2, \ldots, n_m) =$

$= \sum_{i=1}^{m}\sum_{n_i=0}^{\infty} n_i p_i(n_i) = \sum_{i=1}^{m}(1-\rho_i)\sum_{n_i}^{\infty} n_i \rho_i^{n_i} = \sum_{i=1}^{m}\frac{\rho_i}{1-\rho_i}$ (A.10)

Invoking Little's formula, yields average system waiting time:

$W = \frac{L}{\lambda} = \sum_{i=1}^{m}\frac{1}{\mu_i - \lambda} = \sum_{i=1}^{m} W_i$ (A.11) is the sum of waiting times in every $M/M/1$ queue[32].

For interconnected m levels network, ith tier has r_i parallel channels of service rate μ_i allows feedback and feedforward from stage i to j and probability q_{ij}. Also, every stage Poisson arrivals from outside are rate ω_i and components probabilities n_i in tiers i, $i = 1, 2, \dots, m$ is

$$P(n_1, n_2, \dots, n_m) = \prod_{i=1}^{m} p_i(n_i) \quad \text{(A.12)}$$

where $p_i(n_i)$ is each probability component and replacing λ by λ_i with simplifications, we have

$$\sum_{i=1}^{m} q_{i,0}\, \lambda_i = \sum_{i=1}^{m} \omega_i \quad \text{(A.13)}$$

where the total output from the system equals total input to system. Hence, any complex outside Poisson fed network operates like $M/M/r_i$ queues cascade in a stable state. Jackson's theorem shown in equation (A.13) indicates combined input feedback and server outputs are not Poisson. However, equation (A.12) suggests independent network stages operate like $M/M/r_i$ queues with input rate λ_i and service rate μ_i, $i = 1,2, \dots., m$[32].