Short Research Article

Theoretical Determination of the Mass Radii of the Nucleons and Heavier Subatomic Particles

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ABSTRACT

Background: The literature contains numerous values of nucleon charge radii with greater interest in proton. The mean square negative radii are reported for the neutron, the scientific relevance notwithstanding. Only in very few instances was the mass radius of the nucleon investigated.

Methods: Theoretical and computational methods.

Objectives: The objectives of this research are to derive, based mainly on classical model, the equation of the radii of nucleons and other subatomic particles heavier than the nucleons and determine by calculation based on the equation the radii of such particles, and elucidate why results may be different from literature values.

Results and discussion: The results showed expectedly that the mass radii of nucleons and heavier subatomic particles are longer than what seemed to be the preferred proton charge radius. The lengths of the calculated radii increase with increase in rest mass of the subatomic particles whose mass must be ≥ the mass of any nucleon

Conclusion: The equation of the mass radius of any nucleon and heavier subatomic particles was derived. Expectedly the radii differ on the basis of differences in masses of the particles. The difference in mass radii as calculated in this research and reported charge radii in the literature may be due to electron capture leading to greater number of elastic collision with resulting neutrons. Two particles of widely different mass possessing different charge must interact attractively or repulsively if they possess similar charges. Otherwise the deflection of beta–rays and similar particles in an electromagnetic field would be impossible.

Keywords: *Nucleons, heavier subatomic particles, nucleon charge radii, nucleon mass radii.*

1. INTRODUCTION

"Passing a true massage to an audience in very strong impressive vocabulary but not clearly understood cannot be better than the same "gospel" in any field passed on in less impressive but simple and comprehensible language. This is the essence of communication".

The literature is replete with vast amount of data on the proton charge radius and surprisingly the "charge radius" of the nucleon considering what may be described as a layman's understanding of a negative charge radius of the neutron often stated in terms of negative mean square radius in the literature [1, 2]. Unlike in the literature where data on charge radii of the proton and neutron are experimentally generated, the interest in this research entails a theoretical method in which derived equation is used for the calculation of the mass radii of elementary particle. Only the data on physical parameters, such as the masses of different elementary particles are of interest in this research. A different approach is intended. Besides, there is no regret being one of such laymen considering the fact that a charge has its region of influence in three dimensions within which another particle of opposite charge can be attracted and on the other hand be repelled if both particles possess the same charge. This issue of negative squared radius is similar to negative kinetic energy criticised by Mills [3], a scientist with highly advanced mathematical competence. The biological importance of the net charge of an ionised atom or group of atoms cannot be overemphasised. All known forces of molecule– molecule interaction arefunctions of hydrophobicity, electrostatics (polar-polar, polar–charge, charge– polar, charge–charge, van der Waal *etc* [4, 5]. A biochemist, physiologist, biophysicist, and related medical scientist must appreciate the importance of the mitochondria (or chloroplast) in the generation of molecular energy via the electron transport chain where proton carriers and cytochrome system predominate. Appropriate radius of the proton could be relevant in the study of energy production and transduction. All the interaction forces are not without the influence of the electronic structures of the atoms and molecules as the case may be which are under the influence of the nuclear electrostaticenvironment. The range of interaction is likely to be under the influence of total charge of the protons in the nucleus and in particular, the size of the nucleus. Incidentally there has always been keen interest mainly on proton charge radius [1, 6–10], a parameter that can be used to characterise the size of the proton, calculate the transition energies in the hydrogen atom for the definition of fundamental constant, the Rydberg constant in particular, and precision test of quantum electrodynamics dynamics, QED [8].

Despite the electrostatic neutrality of the neutron, its "charge radius" *i.e.* – 0.1161 ± 0.0022 fm^2 [11] has attracted the interest of the core physicist, as stated earlier. This characteristic confers on the neutron an exceptional stability to penetrate other materials, make it capable of traveling greater distance, and it makes objects radioactive. This attribute enables it to be used in medical, academic, and industrial application (www.nrc.gov/radiation - basics). Like electron microscope, the possibility of a neutron microscope is not unlikely (https://www.nistgov/neutron–neutron–microscope). Nonetheless, the effectiveness and perhaps the efficiency of instrumentation, must take into account the mass radius of the hadrons or leptons as may be applicable.

An attempt has been made in the past to determine the mass radius of the nucleon, expected to be different from the charge radius. The work of Hare and Papini[12] gave a calculated gravitational mean square radius of the nucleon otherwise called mass distribution of the nucleon, though it appeared the root mean square result of the latter was given. Elsewhere, in general physics paper, Sha [13] approach gave a longer value of the mass radius of the neutron. Further deliberation on this is reserved for the theoretical section. Therefore, the objectives of this research are to derive, based mainly on classical model, the equation of the radii of nucleons and other subatomic particles heavier than the nucleons and determine by calculation based on the equation the radii of such particles, and elucidate why results may be different from literature values.

2. THEORY

The literature contains pieces of information about the methods explored, theoretically and experimentally, for the calculation of mainly proton charge radius. Those methods and associated mathematical formalism are not within the scope of comprehension and are not relevant to the approach in this research. Einstein's mass–energy equation given as the product of mass defect and the velocity of light has been applied in the determination of the radius of the nucleon. The determination is anchored on the observation that the neutron can disintegrate into proton, electron, and antineutrino published in General science journal and latter, surprisingly in American Journal of Biomedical Science and Research under a different title "Coulomb's law stand to the World of elementary particles in the way that Newton's laws of classical mechanics stand to the macroscopic Worlds" [13]. For the purpose of mass radius determination, the mass defect is the difference in the mass of the neutron and the proton [13].

The theoretical determination of the mass radii of the nucleons and heavier subatomic particles needs the derivation of a suitable equation. The derivation of such equation in this research entails the application of a classical approach in which Newtonian principle dominates and basic de Broglie principle. Two different velocities at two different interparticle distance of approaching particles constitute major additional basis of the derivation.

$$\frac{c^2}{L} = \frac{c_x^2}{L_x} \tag{1a}$$

Where,L_x is the cautiously seen as the interparticle distance when $c_x > c$ (the speed of light), L is the interparticle distance that achieves the equation when c is the speed of light in a vacuum with the implication that $c_x > c$. This may cause objection but scientist seem to have observed particles (neutrinos) moving at speed greater than the speed of light. The following justifies this claim. "But now it seems that researchers working in one of the world's largest physics laboratories, under a mountain in central Italy, have recorded particles travelling at a speed that is supposedly forbidden by Einstein's theory of special relativity" https://www.theguardian.com/science/2011/Sep/22. Mathematical construct such as phase velocity of a wave that may be faster than light may be theoretical in line with this research which is absolutely theoretical. By the way one may ask, is the inverse square law still valid? If valid, then as the distance between two antiparticles (positron-electron pair for instance) closes up or $\rightarrow$ zero, the force of attraction $\rightarrow\infty$ and consequently the velocity $\rightarrow\infty$ (the latter can exceed the speed of light). Furthermore, on the issue of what is forbidden by Einstein's theory of relativity is the challenge posed by the classical electron radius otherwise referred to as Lorentz radius or the Thomson scattering length. The challenge or implication lies in the fact that what is forbidden by Einstein's theory of relativity may be violated as shown in the mathematical expression for classical radius of the electron given below.

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \tag{1b}$$

Equation (1b) is nothing but an expression of electrostatic or Coulomb's law as to imply that at certain interparticle distance, the velocity of one of two particles of unequal mass attains a velocity equal to the velocity of light. Calculation based on Eq. (1b) gives ≈ 2.82 fm. Now, the question is: Does the presence of the velocity of light violate Einstein's theory of special relativity? The presence of Eq. (1b) is intended to buttress what may be considered as an inappropriate application of Coulomb's law for the determination of the radius of elementary particles.

All these questions are not research questions but they are intended to point to the fact that highly elementary particle like leptons may be seen to exceed speed of light in a vacuum in line with the context and situation, natural or unnatural, as the case may be. There is also the parameter called ArthurCompton wavelength given as:

$$\lambda_e = \frac{h}{m_e c} \tag{1c}$$

Thus, the appearance of c or higher parameter, c_x is not out of place considering too, the fact that in the course of electron capture the electron continues its motion towards the nucleus, closing up the interparticle distance and effecting the inverse square law; this is expected to cause velocities >c before capture. The speed of electron exceeding the speed of light under unnatural condition is no longer questionable considering the observation made in favour of this view. This is not to imply that without such pieces of information speed of highly elementary particle like electron cannot exceed the speed of light under simulated (unnatural) condition.

It ought not to be an overemphasis to state that $E = m_x c^2$ is Einstein's mass-energy equation which serves as a fundamental basis for this derivation. Therefore, it is not unlikely that the following could be applicable.

$$E_x = m_x c_x^2 \tag{2}$$

Meanwhile,

$$m_x = \frac{h^3}{c_x^3 L^3 m_e^2} \tag{3}$$

In the equations are Planck constant (h) and mass of the electron (m_e) and m_x is the mass of a particle >m_e. Also,

$$m_x c_x^2 = \frac{e^2 \Phi}{4 \pi \varepsilon_0 \alpha \Gamma_x} \tag{4a}$$

$$= \frac{\Phi e^2}{8 \pi \varepsilon_0 \alpha L} \text{ (Because } \Gamma_x = 2L\text{)} \tag{4b}$$

Where α is the fine structure constant, ε_0 is the permittivity of free space, *e* is the charge of the electron, Γ_x is an interparticle distance that cannot be directly quantified, and $\Phi = m_N/m_e$ where m_N is mass of any nucleon and any particle larger than a nucleon. Substitution of Eq. (3) into Eq. (4b) gives:

$$\frac{h^3 c_x^2}{c_x^3 L^3 m_e^2} = \frac{\Phi e^2}{8 \pi \varepsilon_0 \alpha L} \tag{5a}$$

Simplification of Eq. (5a) gives:

$$\frac{h^3}{c_x L^2 m_e^2} = \frac{\Phi e^2}{8 \pi \varepsilon_0 \alpha} \tag{5b}$$

Taking the square of Eq. (5b) and making c_x^2 subject of the formula gives:

$$c_x^2 = \left(\frac{8 \pi \varepsilon_0 \alpha}{\Phi e^2}\right)^2 \left(\frac{h^3}{m_e^2 L^2}\right)^2 \tag{6}$$

Meanwhile Eq. (1a) is rearranged to give:

$$L_x = \frac{c_x^2 L}{c^2} \tag{7a}$$

However, one can assume that L_x is longer the radius of any nucleon or particle heavier than the latter; this assumption leads to a need to divide Eq. (7a) by a factor, Θ such that, L_x/Θ gives the radius (Γ) of any nucleon or particle bigger than the latter. Thus,

$$\Gamma = \frac{c_x^2 L}{c^2 \Theta} \tag{7b}$$

Substitution of Eq. (6) into Eq. (7b) gives:

$$\Gamma = \left(\frac{8 \pi \varepsilon_0 \alpha}{\Phi e^2}\right)^2 \left(\frac{h^3}{m_e^2 L^2}\right)^2 \frac{L}{c^2} \tag{8a}$$

Simplification of Eq. (8a) gives:

$$\Gamma = \left(\frac{8 \pi \varepsilon_0 \alpha}{\Phi e^2}\right)^2 \left(\frac{h^3}{m_e^2}\right)^2 \frac{1}{L^3 c^2} \tag{8b}$$

As formulated in the manuscript under preparation and derived in appendix section with slight adjustment, *L* is given as:

$$L = \left(\frac{2h}{e}\right)^2 \frac{\varepsilon_0 \pi}{\left(\sqrt[12]{m_N}\right)^5} \tag{9}$$

Taking the cube of Eq. (9) and after simplification gives:

$$L^3 = \left(\frac{2h}{e}\right)^6 (\varepsilon_0 \pi)^3 \frac{1}{m_N^3} \tag{10}$$

Substitution of m_N/m_e in place of Φ (this has been interpreted under Eq. (4b)) and Eq. (10) into Eq. (8b) gives:

$$\Gamma = \left(\frac{8\,\pi\,\varepsilon_0\,\alpha}{\Phi\,\alpha\,e^2}\right)^2 \left(\frac{h^3}{m_e^2}\right)^2 \frac{1}{c^2}\frac{e^6 m_N^3}{(2\,h)^6(\varepsilon_0\,\pi)^3} \tag{11a}$$

$$\Gamma = \frac{m_e^2}{m_N^2}\left(\frac{8\,\pi\,\varepsilon_0\,\alpha}{e^2}\right)^2 \left(\frac{h^3}{m_e^2}\right)^2 \frac{1}{c^2}\frac{e^6\,m_N^3}{(2h)^6\,(\varepsilon_0\,\pi)^3} \tag{11b}$$

Simplification of Eq. (11b) gives the final equation. Thus,

$$\Gamma = \frac{\alpha^2\,e^2\,m_N}{\pi\varepsilon_0\,m_e^2\,c^2} \tag{12}$$

The following postulations may hold. The parameter m_N could stand for the mass of any elementary subatomic (body having finite mass and internal structure but negligible dimensions) particle ≥ the mass of proton or the mass of neutron. Given that$\alpha = \frac{e^2}{2h\,\varepsilon_0 c}$, (12) can be restated as:

$$\Gamma = \frac{e^6\,m_N}{4\,\pi\,\varepsilon_0^3\,m_e^2\,h^2 c^4} \tag{13}$$

For the purpose of emphasis and elucidation, Eq. (13) deserves further analysis which begins with the equation whose form is well known.

$$m_N\,c_x^2 = \frac{e^2}{4\,\pi\,\varepsilon_0\,r_x} \tag{14}$$

Where r_x simply stand for the interparticle distance in line with Coulomb law and m_N is, as stated earlier, the mass of any particle or its antiparticle. It is clear therefore, that r_x cannot necessarily be equated to the radius of the target or bullet particle. Making simple rearrangement gives

$$m_N\,r_x = \frac{e^2}{4\,\pi\,\varepsilon_0\,c_x^2} \tag{15}$$

Dividing Eq. (15) by m_e gives

$$\frac{m_N\,r_x}{m_e} = \frac{e^2}{4\,\pi\,\varepsilon_0\,c_x^2\,m_e} \tag{16}$$

The interpretation of Eq. (16) is that r_x represents such interparticle distance at a given time and space(suitable location) such that the product of the factor m_N/m_e and r_x gives the mass radius of the bigger particle. It is instructive to observe that,

$$r_x = \frac{e^6}{4\,\pi\,\varepsilon_0^3\,m_e\,h^2 c^4} \tag{17}$$

Ultimately, Eq. (16) can be restated as

$$c_{\mathrm{x}}^{2} = \frac{e^{2}}{4\,\pi\,\varepsilon_{0}\,\Gamma\,m_{e}} \tag{18}$$

Since $\Gamma < \approx 2.82$ fm (the presumed classical radius of the electron), c_{x} is expectedly $> c$.

3. MATERIALS AND METHODS

3.1 Materials and Equipment

The research is purely theoretical without any form of experiment. Therefore, no material and equipment were needed.

3.2. Methods

The masses of the fundamental and non-fundamental particles were substituted into the model equation, Eq. (13) for the calculation of the mass radius of each particle whose mass is larger than the mass of an electron.

4. RESULTS AND DISCUSSION

The calculated mass radius of the hadrons, mesons and baryons are shown in Table (1). It is clearly the case that the recorded radii are directly proportional to the masses of the particles. An important observation is that the length of neutron mass radius is longer than proton mass radius made possible by Eq. (13). Now the question is: Should the mass radius of the nucleons in particular be longer than charge radius of the proton in particular given that the charge radius of the nucleon is negative validity or otherwise of the reason given to justify such observation notwithstanding? One need to appreciate that there are also magnetic radii for the nucleons, protons and neutrons with a difference. The proton charge radii in the literature include 0.84 ± 0.05 fm [1], 0.856 fm [2], 0.84 ± 0.0004 fm [6, 8, 14, 15], 0.84087 fm [9]and finally for the purpose of this research, 0.831 ± 0.012 fm [10]. This latter figure for the proton charge radius refutes the claim that 0.84 fm may be the most accurate if lower figures seem to point to a more accurate value. Meanwhile CODATA report shows a value of 0.8847 ± 0.012 fm [8]. The radii of the neutron are often stated as mean square negative radii. Example is: – 0.046 ± 0.006 fm[1].

Table 1. The masses and radii of Baryons

Baryons	**Symbols**	**Mass/exp (–27) kg**	**Radii / fm**
Proton	p^{+}	1.672621777(C)	1.100796221
Neutron	n	1.674927351(C)	1.102313573

Lambda	λ	1.988737554	1.308840290
Sigma plus	Σ^{+}	2.120297998	1.395394871
Sigma neutral	Σ^{0}	2.126002516	1.399177958
Sigma minus	Σ^{-}	2.134559293	1.404809397
Xi neutral	Ξ_{0}	2.344210326	1.542779654
Xi minus	Ξ_{-}	2.354896297	1.549818952

The superscript (C) means that the values were from CODATA recommended values [17]. Other mass values were calculated using the equation of mass–energy equivalence (G eV/c^2) with data in the literature (www.Sciencedirect.com /topics/chemistry/baryons).

The only mass radius of the nucleon discovered in the literature is in the paper by Hare and Papini [12] which showed a value 0.7 fm. Another report is that based on unfamiliar chiral bag model which views the radius of the nucleon to be a bag of radius ≈ 0.8 fm [16]. It appeared the value is for the proton and the nucleon. This however, goes against the results (Table 1) from this research. Matter is any substance that has mass and must therefore occupy a three-dimensional space. This definition is relevant in both elementary school and highly advanced research institute or reputable university. However, one should not ignore the fact that two substances may have the same volume, yet differ in density if one has larger amount of matter than the other. One may therefore, assume that the hadrons exist in their most compact form. The only paradox is that, while for a sphere, the radius is equal to the cube root of a constant multiplied by the cube root of the mass, divide by the cube root of the density, Eq. (13), gives a different scenario. Perhaps, this may be as a result of a mass-energy equivalent approach. Nonetheless, the differences in the radii is justified because, there is no way one can compare the radii of a neutron with an electron or positron. Value such as ≈ 2.82 fm named classical radius of the electron which is much longer than the charge radius of the proton, is nothing but a result which expresses the Coulomb law which can be applied to a proton or its antiproton giving much shorter value.

The main interest in this research is the determination of the mass radius of the nucleon whose value should be vital for the accurate determination of the mass radius of the atomic nucleus of any element. An article entitled “Determining the electron structure”published by Burchell (http://www.alternativephysics.org /book/ElectronicStructure.htm) shows that the measured radius of the proton is 1.11 fm. Based on the liquid drop model Utama *et al* [18] calculated the constant parameter, r_0(= $(3/4\pi\rho_0)^{\square}$ where $\rho_0$is nuclear density) in the equation of nuclear radius, to be 1.15 fm. In this decade, Ivanov and kolicov [19] acknowledge the radii of the proton and neutron as 0.84184 (this is < value indicated in this research) and 1.1 fm respectively. These values including the result given by Sha [13] are definitely similar to the result shown for the two nucleons in Table (1) in this research. Meanwhile, the observed charge radius of the proton found to be much less than mass radius deserves elucidation. The scattering of alpha particles was explored by Rutherford to proof the existence of the nucleus as reviewed elsewhere [20]. The same approach which is not intended to proof the existence of the nucleus is adopted for the determination of proton charge radius. Thus, there is a report that "elastic electron-proton scattering (e-p) and the spectroscopy of hydrogen atoms are the two methods traditionally used to determine the proton charge radius, r_p. In 2010, a new method using, muonic hydrogen atom found a substantial discrepancy compared with previous results, which became known as the 'proton radius puzzle'" [10]. The puzzle remains unresolved despite experimental and theoretical efforts [10].

However, in this research, the greatest puzzle lies in the suggestion that the classical radius of the proton is $\approx$ 2.82 fm. Electrons can be scattered in two ways. When electrons strike an extremely hard neutral surface, they should be scattered without loss of energy if the collision is elastic. On the other hand, an approaching electron towards an antiproton will at a givenappropriate time and interparticle distance come to rest before being repelled with a force defined by Coulomb law. Scattering angles result from interaction (which may be either collisional or repulsive) which occurs at an angle to the straight line between the centres of mass of the two particles. If this was not to be the case, the deflection of the beta rays (β-rays) in an electromagnetic field would have been impossible.

If the Coulomb law is now a "primitive model" on the basis of which the phenomenon of deflection can be elucidated, at all levels, it is rather doubtful if the fact that β-rays are similar to electrons is based on "primitive" assumption. "If a student of an elementary school demands explanation from a teacher who has just stated that the same proton which captures an electron can also scatter electrons receives a shocking bang on a table from the teacher who is only desperate for promotion, such action by the teacher should be seen to be antipedagogy and falls short of the imperatives of educational psychology". Due to the effect of extremely large momentum the collision of antiparticle and its particle leads to mutual annihilation unlike the collision of the electron with the proton. What seems to have played out in the presumed electron-proton (e–p) scattering experiment is that, the protons may have captured electrons yielding neutrons which in the presence of some protons that may not have been hit by the electrons, constitute targets on which other advancing electrons collide elastically. In such scenario, there could be a multiplicity of different e–p interparticle distance. Speculatively, one may not rule out the possibility that antiprotons may have been formed also.

It must be realised too that, any particle bearing charge has its three-dimensional region of influence; the diameter of such region must be longer than twice the mass radius of that particle. The proton or electron cannot be an exception. Recent papers [21–23] have shown that the radii of ground state atom and atomic cations other than hydrogenic atom or ions can be determined. There is no way the charge radii of a proton as known in the literature can be a measure of the size of the nucleus of any atom; it would appear therefore, that the charge radius of the proton is different from the radius of its region of electrostatic influence otherwise the spectroscopic data, ionisation energy and Bohr's radius may be seen to be ridiculously invalid. This is not the case!

According to Edwin Cartlidge (www.physicsworld.com /electron scattering experiment), measurement of the proton radius otherwise called the spatial extent of the proton's electric charge, involves aiming a very narrow beam of electrons at either the gaseous or liquid hydrogen and measuring the tiny deflection of the electrons caused by their interaction with hydrogen nuclei, the protons in this case. It is thus, probable that the nature of deflection may reveal just how far each proton's charge extends into space. If the narrow beam of electrons has sufficient energy greater than the electrostatic energy of attraction of the nuclei for their orbital electrons, they can get much closer to the orbital electrons of the hydrogen atoms; this precludes hitting those electrons because the Coulombic law may not permit such because of similarity in charge. However, sufficient electrostatic repulsive force would have been attained that should set the electrons, the hydrogen electrons and the approaching electrons apart leaving the hydrogen ionised. This scenario should therefore, lead to a stronger attraction of the generated proton for the other incoming narrow beam of electrons.The closer the electrons are to the nuclei the shorter the interfacial distance.

To interact with the positive nuclei implies that the electrons must be drawn (attracted) closer to the nuclei. This means that the greater the force of attraction the closer the electrons become such that the distance between the nuclei and the electrons become shorter leading to what may be seen to be a more accurate size of the proton. This approach means that shorter interparticle (or interfacial) distance than the size reported in the literature [10] may still be achieved. The best approach could have been positron-proton scattering experiment, though the challenge may be how to generate the positron. Thus, once the positron with moderate kinetic energy reaches the limit of the proton's electrostatic field, it must meet resistance and be deflected depending on the angle between a straight line linking their centres of mass and the line of approach to the electrostatic field of the proton.

This section is hereby ended with a food for thought. *"It is observed that one's city is under devastating treat from cosmic rays leaden with very high energy ionising beta rays on a scale greater than the treat of covid-19, within the next one year: Now there are two options open for a "super intelligent" choice. A swift construction with high sense of responsibility, humility and honesty of a device that can be mounted in space that can a) attract the high energy ionising rays for a price of ten million Euros and b) that can deflect the high energy ionising rays back into space for a price of one hundred thousand Euros".*

5. CONCLUSION

The equation of the mass radius of any nucleon and heavier subatomic particles was derived. Expectedly the radii differ on the basis of differences in masses of the particles. The difference in mass radii as calculated in this research and reported charge radii in the literatures may be due to electron capture leading to greater number of elastic collisions with resulting neutrons. Two particles of widely different mass possessing different charge must interact attractively or repulsively if they possess similar charges. Otherwise the deflection of beta–rays and similar particles in an electromagnetic field would be impossible.

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APPENDIX

The derivation of the equation is strictly based on de Broglie principle. One considers two particles, a proton and an electron approaching each other at different velocities less than the velocity of light, the velocity of the proton being much less than the velocity of the electron. Nuclear fission that needs not be observed or nuclear fusion as in the sun can create the condition for such motion. However, it needs to be made clear that motion is not applicable to the proton only; rather it may be applicable to a positron and neutron advancing towards each other. Fractional charge appears strange, but if it enhances elucidation of physical phenomenon or entities, so, let it be for now. “Thanks go to the American (USA), Murray Gell-Mann, who coined the word, quark”. A simple analogy needs to be sought. The fact that a polar molecule such a water molecule is electrically neutral does not imply that at an appropriate spatiotemporal arrangement, the molecules cannot be attracted to each other. There is no orientational motion that takes place without appropriate time and duration and space to do so. Thus with an appropriate orientation, which is probable, the neutron (composed of 2 down quarks bearing a total of –2/3 charges and 1 up quark bearing +2/3 charges) and the positron/electron initially existing under enabling environment can mutually be attracted to each other, though such should be a weak attraction.

At certain interparticle distance the de Broglie wave length may just be a fraction of such distance ℓ; the de Broglie wave length decreases as the velocity of the approaching particle increases but perhaps, not indefinitely. The de Broglie equation is written as

$$m_p = h / a \ell u \quad \text{(A.1)}$$

Where, m_p, h, a, and u are the mass of the proton, Planck constant, a fraction, and velocity of the electron. It is clear that, the de Broglie wavelength $\lambda_{dB} = a\ell$ where $a < 1$. Make $a\ell$ subject of the formula and take its cube to obtain respectively:

$$a \ell = h / m_p u \quad \text{(A.2)}$$

$$a^3 \ell^3 = \frac{h^3}{m_p^3 u^3} \quad \text{(A.3)}$$

Meanwhile, m_p is several times greater than the mass of the electron. Taking the number of times to be ϕ and rearranging Eq. (A.3) gives:

$$a^3 \ell^3 = \frac{h^3}{m_p^2 \, \phi \, m_e \, u^3} \quad \text{(A.4)}$$

Thus,

$$a \, \ell = \frac{h}{u.\sqrt[3]{m_p^2 \, \phi \, m_e}} \quad \text{(A.5)}$$

When the electrostatic energy is equal to twice initial kinetic energy of the advancing particles, then, the

$$m_p u^2 = e^2 / 4 \pi \varepsilon_0 \ell \tag{A.6}$$

The velocity, u is, therefore, given as:

$$u = \frac{e}{\sqrt[2]{4 \pi \varepsilon_0 \ell m_p}} \tag{A.7}$$

Substitution of Eq. (A.7) into Eq. (A.5) gives

$$a\ell = \frac{h.\sqrt[2]{4 \pi \varepsilon_0 \ell m_p}}{e.\sqrt[3]{m_p^2 \phi m_e}} \tag{A.8}$$

Take the square of Eq. (A.8) and simplify to obtain respectively the following:

$$a^2 \ell^2 = \frac{4 \pi \varepsilon_0 \ell m_p h^2}{\left(\sqrt[3]{m_p^2 \phi m_e}\right)^2 . e^2} \tag{A.9a}$$

$$a^2 \ell = \frac{4 \pi \varepsilon_0 m_p h^2}{\left(\sqrt[3]{m_p^2 \phi m_e}\right)^2 . e^2} \tag{A.9b}$$

Recall that ϕm_e is = m_N where m_N may be the mass of any nucleon and any mass greater than the mass of anynucleon. Therefore, substitution into Eq. (A.9b) and rearrangement reproduces Eq. (9) where L which is = $a^2\ell$ is now the interparticle distance when the velocities of approaching particles have increased (but this cannot be indefinite) such that the magnitude of λ_{dB}should also decrease. This is clear because $a \gg a^2$ since a is a fraction.