Review Article

Implications and Applications of Electroweak Quantum Gravity

Abstract: We strongly believe that, the success of any unified model depends on its ability to involve gravity in microscopic models. To understand the mystery of final unification, in our earlier publications, we proposed two bold concepts:1) There exist three atomic gravitational constants associated with electroweak, strong and electromagnetic interactions. 2) There exists a strong elementary charge (e_s) in such a way that its squared ratio with normal elementary charge is close to reciprocal of the strong coupling constant. In this review article we propose that, $(\hbar c)$ can be considered as a compound physical constant associated with electroweak gravity. With these new ideas, an attempt is made to understand and fit nuclear stability, binding energy and quark masses. We wish to emphasize that, nuclear binding energy can be fitted with four simple terms having one unique energy coefficient, $\left(e_s^2/4\pi\varepsilon_0 R_0\right) \cong 10.09$ MeV. With reference to proton-electron mass ratio, Newtonian gravitational constant can be estimated in a verifiable approach. Recently observed 3.5 keV photon seems to be an outcome of annihilation of a new charged small or tiny lepton of rest energy 1.75 keV. By questioning and understanding the integral nature of electron's angular momentum, existence of the three atomic gravitational constants can be understood. By studying the stellar magnetic dipole moments with reference to weak and strong interactions, there is a possibility of confirming the existence of atomic gravitational constants.

Keywords: Four gravitational constants; Compound reduced Planck's constant; Nuclear elementary charge; Strong coupling constant; electroweak fermion;

Nomenclature

1) Newtonian gravitational constant = G_N
2) Electromagnetic gravitational constant= G_e
3) Nuclear gravitational constant = G_s
4) Weak gravitational constant = G_W
5) Fermi's weak coupling constant = G_F
6) Reduced Planck's constant = $\hbar$
7) Speed of light = c
8) Mass of proton = m_p
9) Mass of neutron = m_n
10) Mass of electron = m_e
11) Mass of new electroweak fermion = M_w
12) Normal elementary charge = e
13) Strong nuclear elementary charge = e_s
14) Strong coupling constant = α_s
15) Specific charge ratio of proton = (e_s/m_p)
16) Specific charge ratio of electron = (e/m_e)
17) Ratio of specific charge ratio of proton and electron = s
18) Proton number = Z
19) Mass number = A
20) Neutron number = N
21) Stable mass number = A_s
22) Stable neutron number = N_s
23) Lower stable mass number = $(A_s)_{lower}$
24) Mean stable mass number = $(A_s)_{mean}$
25) Upper stable mass number = $(A_s)_{upper}$
27) Free nucleon number coefficient = f
28) Nuclear binding energy = $B_A = BE$
29) Volume energy coefficient = a_v
30) Surface energy coefficient = a_s
31) Coulombic energy coefficient = a_c
32) Asymmetric energy coefficient = a_a
33) Pairing energy coefficient = a_p
34) Nuclear binding energy at $A_s = B_{A_s}$
35) Up quark mass = m_u
36) Down quark mass = m_d
37) Strange quark mass = m_s
38) Charm quark mass = m_c
39) Bottom quark mass = m_b
40) Top quark mass = m_t
41) Force ratio associated with proton and electron = $X \cong e^2/4\pi\varepsilon_0 G_s m_p m_e$
42) Interaction range = r_x
43) Characteristic ratio associated with charged leptons = $\sqrt{4\pi\varepsilon_0 G_e m_e^2/e^2} \cong \gamma$
44) Mass of charged small lepton= $(m_{xl})^{\pm}$
45) Magnetic moment of proton = μ_p
46) Magnetic moment of electron = μ_e
47) Mass of stellar object = M_X
48) Magnetic moment of stellar objects = μ_X

26) Stability coefficient = $k = 4s$	

1. Introduction to large gravitational coupling constants

According to Quantum Chromodynamics (QCD) [1], strong interaction exhibits two important characteristics, Color or quark confinement and Asymptotic freedom. Color confinement deals with production of hadrons from hadrons without an isolated color charge. Asymptotic freedom deals with reduction of strength of interaction with increasing energy scales at decreasing length scale. Visualizing hadrons as particle level black holes having large nuclear gravitational coupling constant $G_s \approx 10^{39} G_N$. Tennakone, De Sabbata, Gasperini, Abdus Salam, Sivaram and K.P. Sinha [2-5] tried to explain quark confinement with spin-2 massive particles. Based on this 'strong nuclear gravity' and by following Hawking's black hole temperature formula [6], quark-gluon plasma temperature can be understood with a remarkable relation of the form [3], $T_{qg} \approx \hbar c^3 / 8\pi k_B G_s m_n \approx 10^{12}$ °K. Keeping this 'strong nuclear gravity' approach in view, to understand weak interactions, in 2013, Roberto Onofrio [7] introduced a large electroweak gravitational coupling constant. In our 2011 and 2012 papers [8,9] and recently published papers [10-18], we introduced a very large electromagnetic gravitational coupling constant for understanding the basics of final unification.

In this review article, by considering the three atomic gravitational coupling constants, we review our recently published four REFERENCE relations [10] with reference to $(\hbar c)$ and tried to infer the proposed four term semi empirical mass formula with possible physics. In section 8, we tried to fit quark masses. In section 9, we presented many result oriented supporting relations. In sections (10 to 13), we show various practical applications of (G_e, G_s, G_w, G_N).

2. History and current status of nuclear binding energy scheme

With respect to nuclear binding energy and Semi Empirical Mass Formula (SEMF), the inverse problem framework [19], allows to infer the underlying model parameters from experimental observation, rather than to predict the observations from the model parameters. Recently, the ground-state properties of nuclei with Z= 8 to 120 from the proton drip line to the neutron drip line have been investigated using the spherical relativistic continuum Hartree-Bogoliubov (RCHB) theory [20] with the relativistic density functional PC-PK1 (A new parameterization for the nuclear covariant energy density functional with nonlinear point-coupling interaction scheme proposed by fitting to observables of 60 selected spherical nuclei, including the binding energies, charge radii and empirical pairing gaps). In this context, in our recently published paper [10], we emphasized the fact that, physics and mathematics associated with fixing of the energy coefficients of SEMF are neither connected with residual strong nuclear force nor connected with strong coupling constant. N. Ghahramany and team members are constantly working on exploring the secrets of nuclear binding energy and magic numbers in terms of quarks [21,22]. Very interesting point of their study is that - nuclear binding energy can be understood with two or three terms having single energy coefficient of the order of 10 MeV.

3. Motivating concepts/Basic Ideas/Assumptions

Even though celestial objects that show gravity are confirmed to be made up of so many atoms, so far scientists could not find any relation between gravity and the atomic interactions. It clearly indicates that, there is something wrong in our notion of understanding or developing the unified physical concepts. To develop new and workable ideas, we emphasize that,

1) Whether particle's massive nature is due to electromagnetism or gravity or weak interaction or strong interaction or cosmic dust or something else, is unclear.
2) Without understanding the massive nature, it is not reasonable to classify the field created by any elementary particle.
3) All the four interactions seem to be associated with $(\hbar)$.

4) Nobody knows the mystery of $(\hbar)$ which seems to be a basic measure of angular momentum.
5) Nobody knows the mystery of existence, stability and behavior of ‘proton’ or ‘electron’.
6) ‘Mass’ is a basic property of space-time curvature and basic ingredient of angular momentum.
7) Atoms are mainly characterized by protons and electrons.
8) ‘Free neutron’ is an unstable particle.

Based on these 8 points, we propose the following new and workable concepts.

Hypothesis-1: The four basic interactions can be allowed to have four different gravitational constants.

$$G_e \cong 2.374335\times10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$$
$$G_s \cong 3.329561\times10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$$
$$G_w \cong 2.909745\times10^{22}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$$
$$G_N \cong 6.679855\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$$

Hypothesis-2: There exists a strong elementary charge in such a way that its squared ratio with normal elementary charge is close to inverse of the strong coupling constant.

Hypothesis-3: $(\hbar c)$ can be considered as a compound physical constant connecting electroweak interaction.

Hypothesis-4: There exists a characteristic electroweak fermion of rest energy [17], $M_w c^2 \cong 584.725$ GeV.

Hypothesis-5: Fermi’s weak coupling constant (G_F) [23,24] can be considered as the basic unified coupling constant.

With these bold ideas, starting from charged lepton rest masses to stellar masses can be understood. In addition to that, Newtonian gravitational constant can be estimated in a *verifiable approach*. We appeal that, by thoroughly analyzing the bold ideas, it may be possible to understand the combined role of the four gravitational constants in understanding the vector and tensor nature of fundamental forces and their interaction ranges.

4. Characteristic unified relations

Based on the above points, we propose the following new and workable relations.

(1) $(\hbar c)$ can be considered as one of the basic interaction coupling constants associated with quantum phenomena. It can be considered as a compound physical constant,

$$\hbar c \cong G_w M_w^2 \cong \sqrt{G_F\left(\frac{c^4}{4G_w}\right)}$$
$$\Rightarrow \hbar \cong \frac{G_w M_w^2}{c} \cong \sqrt{\frac{G_F c^2}{4G_w}} \qquad (1A)$$

where $\left(\frac{c^4}{4G_w}\right) \cong 6.9401\times10^{10}$ N is the characteristic force associated with electroweak interaction.

$$m_e \cong \left(\frac{G_w}{G_s}\right) M_w \qquad (1B)$$

$$m_p \cong \left(\frac{G_s}{G_w}\right)\left(\frac{G_s}{G_e}\right) M_w \cong \left(\frac{G_s^2}{G_w G_e}\right) M_w \qquad (1C)$$

$$\frac{m_p}{m_e} \cong \frac{G_s^3}{G_w^2 G_e} \qquad (1D)$$

(2) There exists a strong elementary charge (e_s) in such a way that,

$$\left.\begin{aligned} \frac{m_p}{m_e} &\cong \left(\frac{G_s m_p^2}{\hbar c}\right)\left(\frac{G_e m_e^2}{\hbar c}\right) \\ &\cong \left(\frac{e_s^2}{4\pi\varepsilon_0 G_s m_p^2}\right)\Big/\left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right) \end{aligned}\right\} \qquad (2)$$

$$\rightarrow \left\{\begin{aligned} \frac{e_s^2}{e^2} &\cong \left(\frac{G_s m_p^3}{G_e m_e^3}\right) \cong \left(\frac{G_s m_p^2}{\hbar c}\right)^2 \cong \frac{1}{\alpha_s} \\ \frac{e_s}{e} &\cong \sqrt{\frac{G_s m_p^3}{G_e m_e^3}} \cong \left(\frac{G_s m_p^2}{\hbar c}\right) \cong \sqrt{\frac{1}{\alpha_s}} \end{aligned}\right\} \qquad (3)$$

where, $\alpha_s \cong$ Strong coupling constant

Based on these relations,

$$\boxed{\begin{array}{l} e_s \cong 2.9463591e,\ \alpha_s \cong 0.1151937 \\ \text{and } \dfrac{1}{\alpha_s} \cong 8.681032 \end{array}}$$

5. Understanding proton-neutron stability with three atomic gravitational constants

In our recently published paper [10], we proposed the following semi empirical relations (4) to (8) for fitting nuclear stability and binding energy.

With reference to the ratio of specific charge ratio of proton and specific charge ratio of electron,

$$\left.\begin{array}{l} s \cong \left\{ \left(\dfrac{e_s}{m_p} \right) \div \left(\dfrac{e}{m_e} \right) \right\} \cong 0.001605 \\ \cong \sqrt{\dfrac{G_s m_p}{G_e m_e}} \cong \dfrac{G_s m_p m_e}{\hbar c} \cong \dfrac{\hbar c}{G_e m_e^2} \cong \dfrac{G_s^2}{G_e G_w} \cong \dfrac{m_p}{M_w} \end{array}\right\} \quad (4)$$

$$\text{where, } M_w \cong \sqrt{\hbar c / G_w} \cong 584.725 \text{ GeV}/c^2$$

Nuclear mean beta stability line can be explained with a relation of the form,

$$\begin{array}{l} A_s \cong Z + N_s \cong 2Z + s(2Z)^2 \\ \cong 2Z + (4s)Z^2 \cong 2Z + kZ^2 \cong Z(2 + kZ) \\ \text{where } k \cong 4s \cong 0.0064185 \end{array} \quad (5)$$

With this kind of relation, by guessing the proton number, corresponding stable zone nucleon number can be estimated directly. With even-odd corrections and fine tuning the value of k, better understanding is possible. Considering $k \cong 0.00642$ and by considering a simple quadratic equation, relation (5) can be derived.

$$\left.\begin{array}{l} \text{Let, } x = \dfrac{Zk}{2} \\ C = \dfrac{Zk}{2} = \dfrac{Ak}{4} \left(\because \text{Initially, } Z = \dfrac{A}{2} \right) \\ \quad \text{and} \quad x^2 + x - C \cong 0 \\ \left\{ \begin{array}{l} x^2 \text{ coefficient} = 1 \\ x \text{ coefficient} = 1 \end{array} \right\} \\ \quad \to \dfrac{Zk}{2} \cong \dfrac{-1 \pm \sqrt{kA+1}}{2} \\ \Rightarrow Z \cong \dfrac{-1 \pm \sqrt{kA+1}}{k} \cong \dfrac{A}{\left(2.0 + 0.0153 A^{2/3}\right)} \end{array}\right\} \quad (5A)$$

By considering a factor like $\left[2 \pm \sqrt{k}\right]$, likely possible range of A_s can be explained with,

$$\left\{\begin{array}{l} (A_s)_{lower} \cong Z(1.92 + kZ) \\ (A_s)_{mean} \cong Z(2.0 + kZ) \\ (A_s)_{upper} \cong Z(2.08 + kZ) \end{array}\right\} \quad (6)$$

6. Understanding nuclear binding energy

For $(Z \approx 3 \text{ to } 118)$, close to beta stability line, nuclear binding energy can be fitted with,

$$B_{A_s} \cong \left\{ \left(1 - 0.00189\sqrt{ZN_s}\right) A_s - A_s^{1/3} - \left(\frac{Z}{N_s} \right) \right\} 10.09 \text{ MeV}$$

$$\text{where } 0.00189 = \text{New coefficiet} \cong f$$

(7A)

Binding energy per nucleon can be estimated with,

$$\frac{B_{A_s}}{A_s} \cong \frac{1}{A_s} \left\{ \left(1 - 0.00189\sqrt{ZN_s}\right) A_s - A_s^{1/3} - \left(\frac{Z}{N_s} \right) \right\} 10.09 \text{ MeV}$$

(7B)

See Figure 1. Dashed red curve plotted with relations (5) and (7B) can be compared with the green curve plotted with the following standard SEMF (7D).

$$\left.\begin{array}{l} BE \cong (a_v * A_s) - (a_s * A_s^{2/3}) - \left(a_c * \dfrac{Z*(Z-1)}{A_s^{1/3}} \right) \\ \qquad - \left(a_a * \dfrac{(A_s - 2Z)^2}{A_s} \right) \pm \left(\dfrac{a_p}{\sqrt{A_s}} \right) \\ \text{where} \left\{ \begin{array}{l} A_s \cong Z(2 + kZ) \\ a_v \cong 15.78 \text{ MeV};\ a_s \cong 18.34 \text{ MeV};\ a_c \cong 0.71 \text{ MeV}; \\ a_a \cong 23.21 \text{ MeV};\ a_p \cong 12.0 \text{ MeV}; \end{array} \right. \end{array}\right\} \quad (7C)$$

Binding energy per nucleon can be estimated with,

$$\frac{BE}{A_s} \cong \frac{1}{A_s} \left[\begin{array}{l} (a_v * A_s) - (a_s * A_s^{2/3}) - \left(a_c * \dfrac{Z*(Z-1)}{A_s^{1/3}} \right) \\ \qquad - \left(a_a * \dfrac{(A_s - 2Z)^2}{A_s} \right) \pm \left(\dfrac{a_p}{\sqrt{A_s}} \right) \end{array} \right] \quad (7D)$$

For light, medium and heavy atomic nuclide, fit is reasonable.

7. Review on nuclear binding energy scheme

In this section, we try to infer and review relation (7A) for its best possible physics back ground. With further study, nuclear stability and binding energy can be understood with Up and Down quarks.

We propose that,

(1) Nuclear unit radius can be expressed as, $R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.239291$ fm

(2) $B_0 \cong \left(\frac{1}{\alpha_s}\right)\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \cong \frac{e_s^2}{4\pi\varepsilon_0 R_0} \cong \frac{e_s^2}{8\pi\varepsilon_0 \left(G_s m_p/c^2\right)} \cong 10.09$ MeV can be considered as the 'unique' binding energy coefficient. Here, we wish to emphasize the point that, the proposed energy coefficient 10.09 MeV is directly connected with nuclear potential and strong coupling constant whereas energy coefficients of various semi empirical formulae are so chosen to fit the mass data and no way connected with strong interaction. Hence our proposed 10.09 MeV can be called as "unique". With reference to the recommended [24] up quark rest energy of 2.15 MeV and down quark rest energy of 4.7 MeV, it is quite interesting to note that, $\frac{\left[(2m_u + m_d)c^2 + (m_u + 2m_d)c^2\right]}{2} \cong 10.275$ MeV.

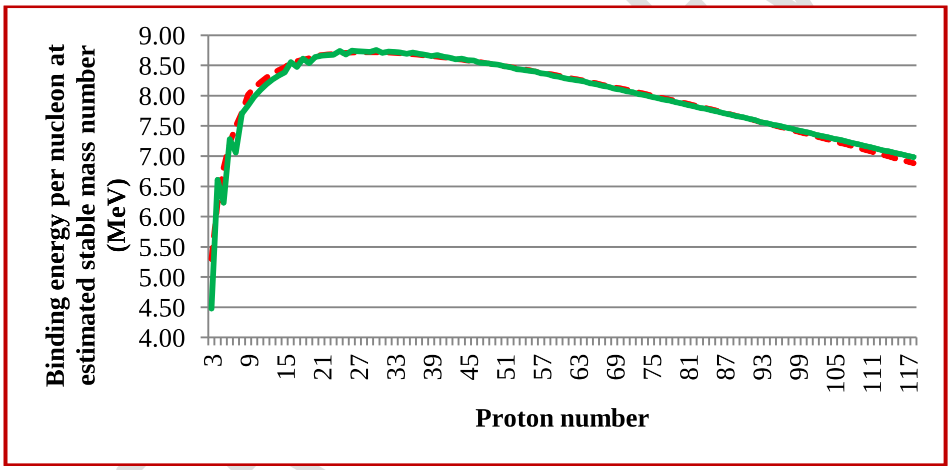

Figure 1: Binding energy per nucleon close to stable mass numbers of Z = 3 to 118

(3) For increasing (Z, A), all nucleons will not involve in nuclear binding energy scheme.

(4) The new numbers $(f \text{ and } k)$ seem to play an interesting role in understanding nuclear stability and binding energy. Both can be considered as the characteristic outcomes of the combined effect of strong and electromagnetic coupling constants. We noticed that, $(k, f) \cong \left(\frac{(1-\alpha_s)^n}{2n-1}\right)\alpha \cong (0.00646, 0.001904)$ where $n \cong 1, 2$; It needs further study.

(5) Nucleons that are not involving in nuclear binding energy scheme can be called as 'free nucleons' and can be represented by $A_f \cong f \times A\sqrt{ZN}$ where the coefficient $f \cong 0.00189$ can be called as 'Free nucleon number coefficient'. With reference to the semi empirical mass formula, quantitatively, $f \cong 2(a_c/a_a)^2 \cong 0.0018753$ where $a_c = 0.71$ MeV and $a_a = 23.21$ MeV.

(6) Nucleons that involve in nuclear binding energy scheme can be called as 'active nucleons' and can be represented by $A_a \cong A - A_f \cong A\left(1 - 0.00189\sqrt{ZN}\right)$.

(7) For $Z = 11$ to 92, when $(A_a - 2Z) \cong 0$, corresponding A seems to represent the possible existence of lower stability line.

(8) The ad-hoc coefficient 0.00189 somehow, seems to lie between $\{s \cong 0.0016 \text{ and } k \cong 0.0064\}$. With reference to electromagnetic interaction, we

consider, $\left[k/\ln(30)\right] \cong 0.00189$ where 30 is a characteristic representation of atomic number below which nuclear binding strength is approximately $[Z/30]^{\sqrt{k}}\left(1/\alpha_s\right) \cong [Z/30]^{0.08} \times 8.68$. From Z=30 onwards, strength of nuclear binding energy remains constant at $\left(1/\alpha_s\right) \cong 8.68$. Based on this concept, for Z = (2 to 118), close to stable mass numbers, binding energy [8] can also be *approximated* with,

$$\left.\begin{array}{l}\text{For } Z < 30 \text{ and } A_s \cong Z\left(2+kZ\right), \\ \left(B_{A_s}\right) \cong \left(\dfrac{Z}{30}\right)^{0.08}\left\{A_s - \left[\left(0.00189 N_s^2\right) + \dfrac{1}{2}\right]\right\} 9.16 \text{ MeV}\end{array}\right\} \tag{20}$$

$$\text{where, }\left\{\begin{array}{l}\dfrac{e_s^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)} - \left(\dfrac{e^2}{4\pi\varepsilon_0 R_0}\right) \cong 8.928 \text{ MeV} \\ \dfrac{e_s^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)} - \dfrac{3}{5}\left(\dfrac{e^2}{4\pi\varepsilon_0 R_0}\right) \cong 9.395 \text{ MeV} \\ \text{and } \dfrac{8.928+9.395}{2} \cong 9.16 \text{ MeV}\end{array}\right\}$$

$$\left.\begin{array}{l}\text{For } Z \geq 30 \text{ and } A_s \cong Z\left(2+kZ\right), \\ \left(B_{A_s}\right) \cong \left\{A_s - \left[\left(0.00189 N_s^2\right) + \dfrac{1}{2}\right]\right\} 9.16 \text{ MeV}\end{array}\right\} \tag{21}$$

(9) Binding energy can be assumed to decrease with increasing radius**.**

(10) Decreasing proton-neutron ratio seems to play an interesting role in increasing binding energy.

(11) Considering isotopes, stable mass number plays an interesting role in estimating the binding energy of other stable and unstable isotopes in the form of $\left(\left(A_s - A\right)^2 / A_s\right)$. This can be considered as a representation of asymmetry about the mean line of stability. It needs further investigation.

(12) In case of Deuteron, there exists no strong interaction between proton and neutron [14,17].

(13) Above and below the stable mass numbers, binding energy can be *approximated* with,

$$B_A \cong \left\{\begin{array}{l}\left(1 - 0.00189\sqrt{ZN}\right)A - A^{1/3} \\ -\left(\dfrac{Z}{N}\right) - \dfrac{\left(A_s - A\right)^2}{A_s}\end{array}\right\} B_0 \tag{8A}$$

Here in this expression, one can find four simple terms. They can be expressed as,

Term-1:

$$\left(1 - 0.00189\sqrt{ZN}\right)A \times B_0$$

Term-2:

$$-A^{1/3} \times B_0$$

Term-3:

$$-\left(\frac{Z}{N}\right) \times B_0$$

Term-4:

$$-\frac{\left(A_s - A\right)^2}{A_s} \times B_0$$

Binding energy estimated with relation (8A) can be compared with the following standard semi empirical mass formula.

$$BE \cong \left\{\begin{array}{l}\left(a_v * A\right) - \left(a_s * A^{2/3}\right) - \left(a_c * \dfrac{Z*(Z-1)}{A^{1/3}}\right) \\ -\left(a_a * \dfrac{(A-2Z)^2}{A}\right) \pm \left(\dfrac{a_p}{\sqrt{A}}\right)\end{array}\right\} \tag{8B}$$

See Figure 2 for the estimated isotopic binding energy of Z=50. Dotted blue curve plotted with relations (5) and (8A) can be compared with the green curve plotted with relation (8B). Based on Figures 1 and 2, it is possible to say that,

a) Relations (5), (7), (8A) and (8B) can also be given some priority in understanding nuclear binding energy scheme.
b) Estimated binding energy can also be compared with spherical Relativistic Continuum Hartree-Bogoliubov (RCHB) theory data [22] and Thomas-Fermi model (Table of nuclear masses, nsdssd.lbl.gov, 1994).
c) For $(N < Z)$ and $(N \approx Z)$ estimated binding energy seems to be increasing compared to SEMF estimation.
d) For $(A >> A_s)$, estimated binding energy seems to be decreasing compared to SEMF estimation.

e) Modifying third and fourth terms into a new term of the form $\left(1+\frac{(A_s-A)^2}{A_s}\right)$ binding energy for $(A << A_s)$ and $(A >> A_s)$ can be understood.

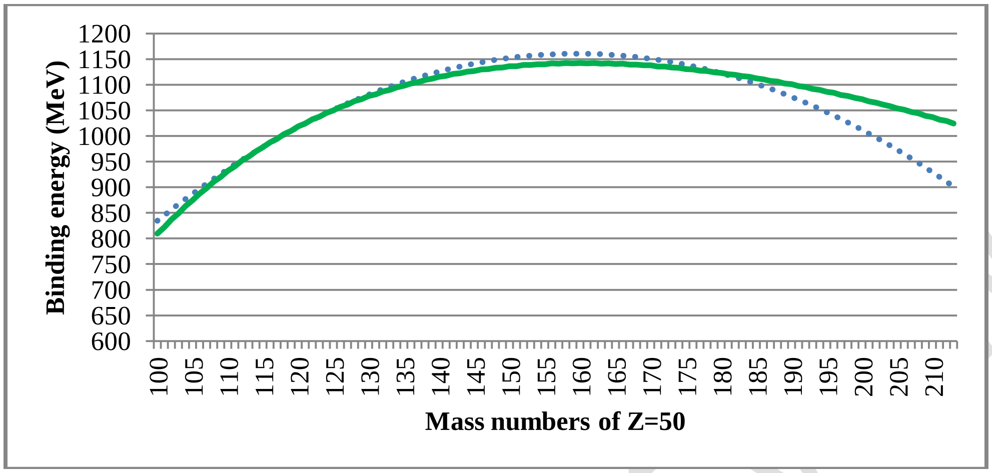

Figure 2: Isotopic binding energy of Z=50

8. To estimate quark masses

In our earlier paper [25], we proposed that,

a) Up, Strange and Bottom quarks are in geometric series.
b) Down, Charm and Top quarks are in another geometric series.

We modify these ideas as:

a) Down and Up quarks are ground state particles and their mass ratio is 2.
b) Strange and Bottom quarks are first generation particles having a geometric series with a geometric ratio of, $g_u \cong \sqrt{m_p/m_e} \cong \sqrt{1836.152} \cong 42.85$.
c) Charm and Top quarks are second generation particles having another geometric series with a geometric ratio of, $g_d \cong (1/\alpha) \cong 137.036$.
d) These two geometric ratios can be fitted with $\frac{e^2}{4\pi\varepsilon_0 G_s m_p m_e} \cong 4.547665$.

Considering Up and Down quarks as ground state particles, quark masses [24] can be fitted in the following way.

Step-1: To fit Up quark mass

$$\left.\begin{aligned} \frac{e^2}{4\pi\varepsilon_0 G_s m_p m_u} &\cong 1 \quad \text{and} \\ \frac{G_s m_p}{c^2} &\cong \frac{e^2}{4\pi\varepsilon_0 (m_u c^2)} \end{aligned}\right\} \tag{9}$$

$$\rightarrow m_u c^2 \cong 2.324 \text{ MeV}$$

Step-2: To fit Down quark mass

$$\left.\begin{aligned} \frac{m_d c^2}{m_u c^2} &\cong 2 \quad \text{and} \\ m_d c^2 &\cong 4.648 \text{ MeV} \end{aligned}\right\} \tag{10}$$

$$\frac{e^2}{4\pi\varepsilon_0 G_s m_p m_d} \cong \frac{1}{2} \tag{11}$$

Based on the estimated up and down quarks, we noticed that,

$$\frac{m_u c^2 + m_d c^2}{m_n c^2} \cong 0.0074256 \cong \alpha \tag{12}$$

where m_n = average mass of nucleon and

α = fine structure constant.

Step-3: To fit Strange and Bottom quark masses

Considering Strange and Bottom quarks as first generation particles, their masses can be fitted with,

$$m_s c^2 \cong \sqrt{\frac{m_p}{m_e}}\left(m_u c^2\right) \cong 99.578 \text{ MeV} \tag{13}$$

$$\left.\begin{aligned} m_b c^2 &\cong \left(\sqrt{\frac{m_p}{m_e}}\right)^2 \left(m_u c^2\right) \\ &\cong \left(\frac{m_p}{m_e}\right)\left(m_u c^2\right) \cong 4266.95 \text{ MeV} \end{aligned}\right\} \tag{14}$$

Step-4: To fit Charm and Top quark masses

Considering Charm and Top quarks as second generation particles, their masses can be fitted with,

$$m_c c^2 \cong 2\left(\frac{1}{\alpha}\right)\left(m_d c^2\right) \cong 1273.806 \text{ MeV} \tag{15}$$

$$m_t c^2 \cong 2\left(\frac{1}{\alpha}\right)^2 \left(m_d c^2\right) \cong 174557.2 \text{ MeV} \tag{16}$$

Step-5: To fit the two geometric ratios

Let,

$$\frac{e^2}{4\pi\varepsilon_0 G_s m_p m_e} \cong 4.547665 \cong X \tag{17}$$

Based on this definition, we noticed that,

$$\left(g_u, g_d\right) \cong \left(\mathrm{e}^x - X\right) \mp \left(\frac{\mathrm{e}^x}{2}\right) \tag{18}$$

$$\left.\begin{aligned} g_u &\cong \left(\mathrm{e}^x - X\right) - \left(\frac{\mathrm{e}^x}{2}\right) \cong 42.6582 \\ g_d &\cong \left(\mathrm{e}^x - X\right) + \left(\frac{\mathrm{e}^x}{2}\right) \cong 137.069852 \end{aligned}\right\} \tag{19}$$

9. Result oriented discussion

(1) With our long experience in this field, we consider the following four relations as REFERENCE relations [10,17,18]. They need further investigation.

$$\left.\begin{aligned} &A)\quad \frac{m_p}{m_e} \cong 2\pi\sqrt{\frac{4\pi\varepsilon_0 G_e m_e^2}{e^2}} \\ &B)\quad \hbar c \cong \frac{\left(G_e^2 G_N\right)^{1/3} m_p^4}{m_e^2} \\ &C)\quad \frac{m_p}{m_e} \cong \left(\frac{G_s}{G_e^{1/3} G_N^{2/3}}\right)^{1/7} \\ &D)\quad \frac{G_w}{G_N} \cong \left(\frac{m_p}{m_e}\right)^{10} \end{aligned}\right\} \tag{22}$$

Note points:

a) Relation (22A) is our given definition for G_e.

b) Inserting (22B) in relation (23), relation (22C) can be obtained.

c) Relation (22D) can be inferred from relation (32).

d) With further study, other possible relations for G_e and other set of REFERENCE relations can also be developed.

(2) The fundamental question to be answered is: How to justify the estimated values of $\left(G_N, G_s, G_w\right)$ with respect to defined G_e? With reference to current literature pertaining to gravity, it seems impossible to answer this question. By considering 'quantum gravity', it seems possible to find an answer. We noticed that,

$$\left.\begin{aligned} \left(\frac{G_s m_p}{c^2} \Big/ \sqrt{\frac{G_N \hbar}{c^3}}\right) &\cong \left(\frac{m_p}{m_e}\right)^6 \\ \left(\frac{G_s m_p}{c^2} \Big/ \sqrt{\frac{G_N \hbar}{c^3}}\right)^{\frac{1}{6}} &\cong \left(\frac{m_p}{m_e}\right) \end{aligned}\right\} \tag{23}$$

In a simplified form,

$$\frac{G_s^2 m_p^2}{G_N \hbar c} \cong \left(\frac{m_p}{m_e}\right)^{12} \tag{24}$$

a) Considering CODATA-2014 (Committee on Data for Science and Technology) recommended big $G \equiv G_N$, with relation (24) value of G_s can be estimated.

$$G_s \cong \left(\frac{m_p^5}{m_e^6}\right)\sqrt{G_N \hbar c} \tag{25}$$

b) By inserting the values of $\left(G_N, G_s\right)$ in relation (22C), value of G_e can be estimated.

$$G_e \cong \left(\frac{m_e}{m_p}\right)^{21} \frac{G_s^3}{G_N^2} \quad (26)$$

c) By inserting the value of G_e in relation (22A), value of $\left(m_p/m_e\right)$ can be estimated.

$$\left.\begin{array}{l} A)\ \left(\frac{m_p}{m_e}\right) \cong 2\pi\sqrt{\frac{4\pi\varepsilon_0 G_e m_e^2}{e^2}}\ (\text{Or}) \\ B)\ \left(\frac{m_p}{m_e}\right) \cong \sqrt[4]{2\pi\sqrt{\frac{4\pi\varepsilon_0 \hbar^{3/2} c^{3/2}}{e^2 G_N^{1/2} m_e}}} \end{array}\right\} \quad (27)$$

Calculating the %error in the estimated $\left(m_p/m_e\right)$, error in G_N can be reviewed. %error means –

$$\left(\frac{\text{Recom.value of}\left(m_p/m_e\right) - \text{Estim.value of}\left(m_p/m_e\right)}{\text{Recom.value of}\left(m_p/m_e\right)}\right)\times 100$$

Based on relation (27A) estimated $\left(m_p/m_e\right) \cong 1836.549774$ is 216 ppm higher than the recommended $\left(m_p/m_e\right)$.

A note about 'ppm': It is a commonly accepted relative notation of representation of any measured quantity expressed as number of Parts Per Million.

d) Based on relation (27B), estimated $\left(m_p/m_e\right) \cong 1836.251941$ is 54 ppm higher than the recommended $\left(m_p/m_e\right)$. It is surprising to note that, independent of all the proposed three atomic gravitational constants,

$$\begin{aligned} G_N &\cong \left(\frac{m_e}{m_p}\right)^{14} \left(\frac{4\pi\varepsilon_0 \hbar c}{e^2}\right)^2 \left(\frac{16\pi^4 \hbar c}{m_p^2}\right) \\ &\cong \left(\frac{m_e}{m_p}\right)^{14} \left(\frac{1}{\alpha}\right)^2 \left(\frac{16\pi^4 \hbar c}{m_p^2}\right) \quad (28) \\ &\cong 6.679855429\times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \end{aligned}$$

where, $\alpha \cong$ Fine structure constant

$$\left.\begin{array}{l} A)\ \left(\frac{m_p}{m_e}\right) \cong \left\{\frac{4\pi^2}{\alpha}\sqrt{\frac{\hbar c}{G_N m_p^2}}\right\}^{1/7} \\ B)\ \alpha \cong 4\pi^2 \left(\frac{m_e}{m_p}\right)^7 \sqrt{\frac{\hbar c}{G_N m_p^2}} \end{array}\right\} \quad (29)$$

Based on relation (29), by estimating the error in $\left(\frac{m_p}{m_e}\right)$ or α, G_N value can be reviewed.

e) By considering $\left(m_p/m_e\right)$ and $\left(\alpha\right)$ as key tools, actual G_N value seems to be higher than the CODATA and other big G experimental values. With further study and considering other possible relations for G_e and repeating the above steps, G_N can be refined.

f) Applying the concept of Penrose model of extraction of black hole energy [26] to a proton, it is also possible to show that [27],

$$\left(1-\frac{1}{\sqrt{2}}\right)\frac{m_p}{m_e} \cong \left(\frac{\hbar c}{G_N m_p^2}\right)^{1/14} \quad (30)$$

where,

$$\left\{\frac{4\pi^2}{\alpha}\right\}^{\frac{1}{7}} \cong 3.414393 \cong 2+\sqrt{2} \cong \left(1-\frac{1}{\sqrt{2}}\right)^{-1}$$

$$\begin{aligned} G_N &\cong \left[\left(1-\frac{1}{\sqrt{2}}\right)\frac{m_p}{m_e}\right]^{-14}\left(\frac{\hbar c}{m_p^2}\right) \quad (31) \\ &\cong 6.674946866\times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \end{aligned}$$

This estimated value is 130 ppm higher than the, CODATA-2014, recommended value. At the same time, it seems to lie in between experiment carried out by HUST-AAF-2018 result of 6.674484×10^{-11} m^{3}kg^{-1}sec^{-2} and BIPM-2014 result of 6.67554×10^{-11} m^{3}kg^{-1}sec^{-2}. See Table 1 for the historical results [28-35] of G_N.

Table 1: Various experimental values of G_N

Experiment/Year	$G_N/10^{-11}$ m^{3}kg^{-1}sec^{-2}
NIST-1982	6.67248
TR&D-1996	6.6729
LANL-1997	6.67398
UWash-2000	6.674255
BIPM-2001	6.67559

UWup-2002	6.67422
MSL-2003	6.67387
HUST-2005	6.67222
UZur-2006	6.67425
HUST-2009	6.67349
JILA-2010	6.6726
BIPM-2014	**6.67554**
LENS-2014	6.67191
UCI-2014	6.67435
HUST-TOS-2018	6.674184
HUST-AAF-2018	**6.674484**

About HUST-AAF-2018: It is an experiment designed to measure big G with Angular-Acceleration Feedback (AAF) method performed at Huazhong university of science & technology, Hongshan District, Wuhan, Hubei province, China.

About BIPM: The **International Bureau of Weights and Measures (IBWM)** (French: *Bureau international des poids et mesures (BIPM)*) is an intergovernmental organization that was established by the Metre Convention, through which member states act together on matters related to measurement science and measurement standards (i.e. the International System of Units). The organization is commonly referred to by its French initialism, BIPM. The BIPM's secretariat and formal meetings are housed in the organizations headquarters in Sevres, France.

(3) Fermi's weak coupling constant is one of the most critical and complicated nuclear physical constants. It can be approximated as,

$$\left.\begin{aligned} G_F &\cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \\ &\cong \left[\left(G_e^2 G_N\right)^{1/3} m_p^2\right]\left(\frac{2G_s m_p}{c^2}\right)^2 \\ &\cong \left(\frac{m_p}{m_e}\right)^{10} \frac{4G_N \hbar^2}{c^2} \cong \frac{4G_w \hbar^2}{c^2} \\ &\cong 1.44021048\times10^{-62}\ \text{J.m}^3 \end{aligned}\right\} \quad (32)$$

This estimated value is 3036 ppm higher than the recommended value, $G_F \cong 1.435850984\times10^{-62}$ J.m^3 where $G_F/(\hbar c)^3 \cong 1.1663787\times10^{-5}\,\text{GeV}^{-2}$ [31].

(3) The basic question to be answered is: How do the invoked four gravitational constants address the issues pertaining to vector forces of electromagnetism, tensor forces of gravity and vector-axial vector forces of weak interaction and gluons of strong interaction? It needs further study with respect to the four gravitational constants and the compound $(\hbar)$. In this context, we could notice that, 'Range of four interactions' can be expressed by a model relation of the form [27],

$$r_x \approx \frac{M_x}{m_p}\sqrt{\frac{G_x \hbar}{c^3}} \quad (33)$$

where M_x, G_x, m_p represent the characteristic mass of interaction, characteristic gravitational constant and proton mass respectively. As most of the atomic matter is characterised by protons, this relation can be given some consideration.

a) Strong interaction range,

$$r_s \approx \sqrt{G_s \hbar / c^3} \approx 0.361\times10^{-15}\ \text{m} \quad (34)$$

b) Weak interaction range,

$$\begin{aligned} r_w &\approx \left(80400\ \text{MeV}/938.272\ \text{MeV}\right)\sqrt{G_w \hbar/c^3} \\ &\approx 2.892\times10^{-17}\ \text{m} \end{aligned} \quad (35)$$

c) Electromagnetic interaction range at atomic level,

$$\begin{aligned} r_{em} &\approx \left(931.5\ \text{MeV}/938.272\ \text{MeV}\right)\sqrt{G_e \hbar/c^3} \\ &\approx 9.57\times10^{-12}\ \text{m} \approx 9.57\ \text{pm} \end{aligned} \quad (36)$$

d) Gravitational interaction range for Sun,

$$\begin{aligned} r_{sun} &\approx \left(2.0\times10^{30}\text{kg}/1.672\times10^{-27}\text{kg}\right)\sqrt{G_N \hbar/c^3} \\ &\approx 1.933\times10^{22}\ \text{m} \end{aligned} \quad (37)$$

10. Specific unified relations connected with nuclear radius and Bohr radius

Characteristic Schwarzschild radius of proton and Schwarzschild radius of atom can be addressed with the following relations.

$$\begin{aligned} R_p &\cong \frac{2G_s m_p}{c^2} \cong 1.2393\ \text{fm} \\ &= \text{Characteristic nuclear charge radius [36,37]} \end{aligned} \quad (38)$$

$$\begin{aligned} R_{(Z,A)} &\cong \left\{Z^{1/3} + \left(\sqrt{Z(A-Z)}\right)^{1/3}\right\}\left(\frac{G_s m_p}{c^2}\right) \\ &= \text{Nuclear charge radius [38]} \end{aligned} \quad (39)$$

$$a_0 \cong \left(\frac{4\pi\varepsilon_0 G_e m_e^2}{e^2}\right)\left(\frac{G_s m_p}{c^2}\right) \cong 5.2918\times10^{-11}\ \text{m} \tag{40}$$

= Bohr radius of Hydrogen atom [39,40]

11. Specific unified relations connected with proton-electron mass ratio

With reference to electroweak interaction,

$$R_w \cong \frac{2G_w M_w}{c^2} \cong 6.7494\times10^{-19}\ \text{m} \tag{41}$$

= Schwarzschild radius of M_w

$$\frac{R_p}{R_w} \cong \left(\frac{2G_s m_p}{c^2}\right) \div \left(\frac{2G_w M_w}{c^2}\right) \cong \frac{G_s m_p}{G_w M_w} \cong \left(\frac{m_p}{m_e}\right) \tag{42}$$

With reference to $R_w \cong 6.7494\times10^{-19}$ m and cconsidering $\left(\frac{m_p}{m_e}\right)$ as a geometric ratio, nuclear radius and atomic radius can be estimated in the following way.

$$R_1 \cong \left(\frac{m_p}{m_e}\right)\left(\frac{2G_w M_w}{c^2}\right) \cong 1.2393\ \text{fm} \tag{43}$$

$$R_2 \cong \left(\frac{m_p}{m_e}\right)^2 \left(\frac{2G_w M_w}{c^2}\right) \cong 2.275\ \text{pm} \tag{44}$$

With reference to electromagnetic gravitational constant, Schwarzschild radius of electron can be addressed with,

$$R_e \cong \left(\frac{2G_e m_e}{c^2}\right) \cong 0.48\ \text{nm} \tag{45}$$

Based on relations (44) and (45) and identifying R_2 and R_e as characteristic length scales associated with characteristic atomic radius, we noticed that,

$$\sqrt{R_2 R_e} \cong \left(\frac{2\sqrt{G_e G_s}\, m_p}{c^2}\right) \cong 33.1\ \text{pm} \tag{46}$$

$\cong R_{atom} \cong$ Scwarzschild radius of atom [41]

12. Applications of G_e in elementary particle physics and astrophysics

A) Understanding the recently observed 3.5 keV galactic photon

Recent galactic X-ray studies [42,43] strongly confirmed the existence of a new photon of energy 3.5 keV. So far, its origin is unknown and unclear. In this context, we propose [15] the following alternative mechanism for understanding the origin of 3.5 keV photon.

1) There exists a characteristic charged small lepton of rest mass,

$$(m_{xl})^{\pm} \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0 G_e}} \cong 1.75\ \text{keV}/c^2 \tag{47}$$

2) With pair annihilation mechanism, (m_{xl}) generates a photon of rest energy 3.5keV
3) With current and future particle accelerators, $(m_{xl})^{\pm} \cong 1.75\ \text{keV}/c^2$ can be generated.

B) Fitting Muon and Tau rest masses

Experimentally observed [24] Muon and Tau rest masses can be fitted in the following way.

$$m_{(\mu,\tau)}c^2 \cong \left\{\left[\gamma^3 + \left(n^2\gamma\right)^n \left(\frac{G_e}{G_N}\right)^{1/4}\right]^{\frac{1}{3}} 1.75\ \text{keV}\right\} \tag{48}$$

where,

$$\gamma \cong \sqrt{\frac{4\pi\varepsilon_0 G_e m_e^2}{e^2}} \cong 292.187 \text{ and } n = 1 \text{ and } 2$$

For $n=1$, obtained $m_\mu c^2 \cong 106.5$ MeV

$n=2$, obtained $m_\tau c^2 \cong 1781.5$ MeV.

At $n=3$, a new heavy charged lepton of rest energy 42.2 GeV can be predicted.

13. Specific unified relations connected with stellar mass limits

With reference to strong nuclear gravitational constant and astrophysics point of view, by considering nucleon as a characteristic building block, stellar mass limit [44,45] can be understood with a relation of the form,

$$\frac{G_N M_X}{G_s m_n} \cong \sqrt{\frac{G_s}{G_N}} \tag{49}$$

Thus, characteristic stellar mass limit can be estimated with a very simple relation of the form,

$$M_X \cong \left(\frac{G_s}{G_N}\right)^{\frac{3}{2}} (m_n) \cong 9.37 \text{ solar masses} \tag{50}$$

Another interesting relation is,

$$\frac{G_N M_X}{G_s \sqrt{m_n M_w}} \cong \sqrt{\frac{G_s}{G_N}} \tag{51}$$

$$M_X \cong \left(\frac{G_s}{G_N}\right)^{\frac{3}{2}} \sqrt{m_n M_w} \cong 234 \text{ solar masses} \tag{52}$$

With reference to electromagnetic gravitational constant, mass limits of super massive stellar objects can be understood.

14. To understand the integral nature of electron's angular momentum

Without considering the rest mass of proton, Bohr's theory of Hydrogen atom [40] attempts to explain the discrete spectral lines. On a whole,

a) If hydrogen atom is characterized by its central mass and central charge,
b) If mass of proton is 1836 times heavier than electron,

then, ignoring proton mass in the calculation of emitted spectral lines seems to be a fundamental snag. Probably it may be the root cause of failure of developing a unified model. With our approach, it is possible to show that,

$$\hbar c \cong G_s M_w m_e \cong \left(\frac{G_w G_e}{G_s}\right) m_p m_e \tag{53}$$

$$\hbar \cong \left(\frac{1}{c}\right)\left(\frac{G_w G_e}{G_s}\right) m_p m_e \tag{54}$$

As per the Bohr's second postulate,

$$(m_e vr) \cong n\hbar \cong n\left(\frac{1}{c}\right)\left(\frac{G_w G_e}{G_s}\right) m_p m_e \tag{55}$$

where, $n = 1, 2, 3, ..$

It can be inferred as,

$$m_e (vr) \cong \left[\left(\frac{1}{c}\right)\left(\frac{G_w G_e}{G_s}\right)(nm_p)\right] m_e \tag{56}$$

Possible interpretation seems to be,

$$\left.\begin{aligned} (vr) &\propto \left(\frac{1}{c}\right) \\ &\propto \left(\frac{G_w G_e}{G_s}\right) \\ &\propto (nm_p) \end{aligned}\right\} \tag{57}$$

Clearly speaking, integral nature of m_p i.e. $m_p, 2m_p$, $3m_p, .. nm_p$, seems to be responsible for the integral nature of electron's angular momentum. This explanation seems to be very natural and very simple. We are working in this direction and planning to seek experts' opinion.

15. To confirm the existence of (G_e, G_w, G_s)

1) At atomic scale, based on relation (46), there is a possibility of confirming the existence of (G_e, G_s).
2) Neutron life time point of view, there is a possibility of confirming the existence of (G_e, G_w). Interesting point to be noted is that by considering moving neutrons and their relativistic mass expression, results of bottle and beam methods can be correlated [46,47,48]. Its characteristic relation is,

$$t_n \cong \left(\frac{G_e^2 m_n^2}{G_w (m_n - m_p) c^3}\right) \cong 874.94 \text{ sec} \tag{58}$$

3) Astrophysics point of view, based on relation (49), there is a possibility of confirming the existence of (G_s).
4) Galactic observations of point of view, based on relation (47), there is a possibility of confirming the existence of (G_e). We emphasize the point that, compared to the concept of decay of cosmic neutrinos, annihilation of $(m_{xl})^{\pm}$ can be given a priority in understanding the origin of 3.5 keV photons. With particle accelerators, existence of $(m_{xl})^{\pm}$, can be confirmed and thereby existence of (G_e) can also be confirmed.
5) With reference to $T \approx \hbar c^3 / 8\pi k_B (G_s m_p)$ or $T \approx \hbar c^3 / 8\pi k_B (G_e m_e)$ or $T \approx \hbar c^3 / 8\pi k_B (G_w m_w)$ and with particle accelerator experiments and by

considering the melting points of nucleons, electrons and other characteristic elementary particles, existence of $(G_e, G_{w,} G_s)$ can be understood.

6) With reference to magnetic dipole moments of elementary particles and stellar objects, there is a possibility of confirming the existence of $(G_{w,} G_s)$

a) Proton magnetic moment can be estimated with a relation of the form,

$$\left.\begin{array}{l}\mu_p \cong \dfrac{e_s\hbar}{2m_p} \cong \dfrac{eG_s m_p}{2c} \\ \cong 1.488\times10^{-26}\ \text{A.m}^2\end{array}\right\} \quad (59)$$

b) Electron magnetic moment can be estimated with a relation of the form,

$$\left.\begin{array}{l}\mu_e \cong \dfrac{e\hbar}{2m_e} \cong \dfrac{eG_s M_w}{2c} \\ \cong 9.274\times10^{-24}\ \text{A.m}^2\end{array}\right\} \quad (60)$$

Since, $G_s m_e \cong G_w M_w$, modifying relation (60) leads to,

$$\left.\begin{array}{l}\mu_e \cong \left(\dfrac{M_w}{m_e}\right)^2 \dfrac{eG_w m_e}{2c} \\ \cong 9.274\times10^{-24}\ \text{A.m}^2\end{array}\right\} \quad (61)$$

c) If mass of stellar object is M_X, with reference to relation (61), stellar magnetic dipole moments [49,50], can be expressed as,

$$\left.\begin{array}{l}\mu_X \cong \left[1+\left(\dfrac{M_w}{M_X}\right)^2\right]\left(\dfrac{eG_w M_X}{2c}\right) \cong \left(\dfrac{eG_w M_X}{2c}\right) \\ \text{since,} \left(\dfrac{M_w}{M_X}\right) << 1 \text{ and } \left[1+\left(\dfrac{M_w}{M_X}\right)^2\right] \cong 1\end{array}\right\} \quad (62)$$

Since any stellar object mass is mainly influenced by nucleon mass, based on relations (59) and (62) and based on the known and unknown internal structural or binding forces, lower, upper and mean values of stellar magnetic moments can be expressed as,

$$\left.\begin{array}{l}(\mu_X)_{lower} \cong \left(\dfrac{eG_w M_X}{2c}\right) \\ (\mu_X)_{upper} \cong \left(\dfrac{eG_s M_X}{2c}\right) \\ (\mu_X)_{mean} \cong \left(\dfrac{e\sqrt{G_w G_s} M_X}{2c}\right)\end{array}\right\} \quad (63)$$

With reference to strong and weak interactions, it is possible to say that,

i. If stellar object is strongly bound, then it can have a maximum magnetic moment and can be expressed by $\left(\dfrac{eG_s M_X}{2c}\right)$.

ii. If stellar object is weakly bound, then it can have a minimum magnetic moment and can be expressed by $\left(\dfrac{eG_w M_X}{2c}\right)$.

iii. To a very rough approximation, mean magnetic dipole moment can be approximated with a relation of the form, $\left(\dfrac{e\sqrt{G_w G_s} M_X}{2c}\right)$.

iv. Earth's estimated mean magnetic dipole moment is 4.986×10^{22} A.m^2 and actual value 8×10^{22} A.m^2.

v. Sun's, estimated mean magnetic dipole moment is 1.65×10^{28} A.m^2 and actual value 3.5×10^{29} A.m^2.

vi. With reference to other stellar compact objects and black holes, further study can be carried out.

16. General discussion

We appeal that,

(1) Success of any unified model depends on its ability to involve gravity in microscopic models.

(2) Full-fledged implementation of gravity in microscopic physics must be able to:

a) Estimate the ground state elementary particle rest masses of the three atomic interactions.

b) Estimate the coupling constants of the three atomic interactions.

c) Estimate the range of all interactions.

d) Estimate the Newtonian gravitational constant.

(3) As the root is unclear and unknown, to make it success or to have a full-fledged implementation, one may be forced to consider a new path that may be out-of-scope of the currently believed unsuccessful unified physics.

(4) In our approach,

a) We assign a different gravitational constant for each basic interaction.

b) We consider proton and electron as the two characteristic building blocks of the four basic interactions.

c) Finally, by eliminating the three atomic gravitational constants, we develop a characteristic relation for estimating the Newtonian gravitational constant. Based on relations (1) and (29), it is possible to show that,

$$\frac{m_p}{m_e} \cong \frac{G_w^{13/4} G_e^{3/2}}{G_s^{9/2} G_N^{1/4}} \tag{64}$$

$$G_N \cong \left(\frac{m_e}{m_p}\right)^4 \frac{G_w^{13} G_e^{6}}{G_s^{18}} \tag{65}$$

d) During this journey, without considering arbitrary numbers or coefficients, we come across many strange and interesting relations for estimating other atomic and nuclear coupling constants.

(5) We strongly believe that, with further study, research and synthesizing the noticed relations in a systematic approach, actual essence of final unification can be understood in a theoretical and experimental approach [51]. In addition to that, magnitudes of elementary physical constants can be reviewed in a unified approach.

(6) During cosmic evolution [52], if one is willing to give equal importance to Higgs boson and Planck mass in understanding the massive origin of elementary particles, then it seems quite logical to expect a common relation in between Planck scale and Electroweak scale [53,54,55].

17. Conclusion

By implementing four such gravitational constants in String theory models, it may be possible to explore the hidden unified physics connected with compound $(\hbar c)$, different forms of fundamental forces and their interaction ranges. Even though derivational procedure is missing, consequences of the proposed four reference relations (22A, 22B, 22C and 22D) seem to be quite interesting and logical. Based on relations (1A,1B,1C and 1D) and with further study, research and confirming the existence of $M_w c^2 \cong 584.725$ GeV, actual essence of final unification can be understood. Independent of large numbers, gap between nuclear scale and Planck scale can be understood via relations like (23) and (33) with proper physics. Considering the applications proposed in sections (10 to 15), to some extent, existence of (G_e, G_s, G_w) can be validated. Finally, theoretical value of G_N can be defined as a standard reference for future nuclear, atomic and gravitational experiments.

Understanding nuclear binding energy with single energy coefficient in terms of fundamental interactions is a very challenging task. In this context, we tried our level best in presenting a very simple and effective semi empirical formula with one unique energy coefficient. It needs further investigation.

References

[1] Frank Wilczek. QCD made simple (PDF). Physics Today. 53 (8): 22–28 (2000)

[2] K. Tennakone, Electron, muon, proton, and strong gravity. Phys. Rev. D, 10, 1722 (1974)

[3] C. Sivaram and K. Sinha, Strong gravity, black holes, and hadrons. Physical Review D., 16(6), 1975-1978 (1977)

[4] De Sabbata V and M. Gasperini. Strong gravity and weak interactions. Gen. Relat. Gravit. 10, 9, 731-741, (1979)

[5] Salam A, Sivaram C. Strong Gravity Approach to QCD and Confinement. Mod. Phys. Lett., v. A8(4), 321–326 (1993)

[6] Hawking, S.W. Particle Creation by Black Holes. Communications in Mathematical Physics, 43, 199-220(1975)

[7] Roberto Onofrio. On weak interactions as short-distance manifestations of gravity. Modern Physics Letters A 28, 1350022 (2013)

[8] Seshavatharam UVS, Lakshminarayana S. To confirm the existence of atomic gravitational constant. Hadronic Journal.34(4):379 (2011)

[9] Seshavatharam UVS, Lakshminarayana S. Molar electron mass and the basics of TOE. Journal of Nuclear and Particle Physics. 2(6):132 -141 (2012)

[10] Seshavatharam UVS and Lakshminarayana S. On the Role of Squared Neutron Number in

Reducing Nuclear Binding Energy in the Light of Electromagnetic, Weak and Nuclear Gravitational Constants – A Review. Asian Journal of Research and Reviews in Physics, 2(3): 1-22, (2019)

[11] Seshavatharam UVS et al. Understanding the constructional features of materialistic atoms in the light of strong nuclear gravitational coupling. Materials Today: 3/10PB, Proceedings 3, 3976-3981 (2016)

[12] Seshavatharam UVS and Lakshminarayana S. Towards a workable model of final unification. International Journal of Mathematics and Physics 7(1), 117-130 (2016)

[13] Seshavatharam UVS and Lakshminarayana S. Understanding the basics of final unification with three gravitational constants associated with nuclear, electromagnetic and gravitational interactions. Journal of Nuclear Physics, Material Sciences, Radiation and Applications 4(1),1-19 (2017)

[14] Seshavatharam UVS and Lakshminarayana S. On the role of 'reciprocal' of the strong coupling constant in nuclear structure. Journal of Nuclear Sciences, 4(2), 31-44 (2017).

[15] Seshavatharam UVS and Lakshminarayana S. Quantum gravitational applications of nuclear, atomic and astrophysical phenomena. Unified Field Mechanics II: Formulations and Empirical Tests, pp. 119-126 (2018)

[16] Seshavatharam UVS and Lakshminarayana S. A virtual model of microscopic quantum gravity, Prespacetime Journal, 9(1), 58-82 (2018).

[17] Seshavatharam UVS and Lakshminarayana S. On the role of four gravitational constants in nuclear structure. Mapana Journal of Sciences, 18(1), 21-45 (2019)

[18] Seshavatharam UVS and Lakshminarayana S. On the Role of Large Nuclear Gravity in Understanding Strong Coupling Constant, Nuclear Stability Range, Binding Energy of Isotopes and Magic proton numbers – A Critical Review. J. Nucl. Phys. Mat. Sci. Rad. A. 6(2), 142–160 (2019)

[19] Cht. Mavrodiev S, Deliyergiyev MA. Modification of the nuclear landscape in the inverse problem framework using the generalized Bethe-Weizsäcker mass formula. Int. J. Mod. Phys. E27: 1850015 (2018)

[20] Xia XW, et al. The limits of the nuclear landscape explored by the relativistic continuum Hatree-Bogoliubov theory. Atomic Data and Nuclear Data Tables. 121-122: 1-215 (2018)

[21] Ghahramany N et al. New scheme of nuclide and nuclear binding energy from quark-like model. Iranian Journal of Science & Technology. A3: 201-208 (2011)

[22] Ghahramany N et al. New approach to nuclear binding energy in integrated nuclear model. Journal of Theoretical and Applied Physics. 6:3 (2012)

[23] Fermi E. Tentativo di una teoria dei raggi β. La Ricerca Scientifica (in Italian). 2 (12), (1933).

[24] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

[25] Seshavatharam UVS and Lakshminarayana S. Super symmetry in strong and weak interactions, Int. J. Mod. Phys. E 19 (2), 263-280 (2010)

[26] R. Penrose and R. M. Floyd, Extraction of Rotational Energy from a Black Hole. Nature Physical Science 229, 177 (1971)

[27] Seshavatharam UVS and Lakshminarayana S. On the role of Newtonian gravitational constant in estimating Proton-Electron mass ratio and baryon mass spectrum. International Journal of Innovative Studies in Sciences Engineering Technology. Vol 5, Issue 10, 18-21 (2019)

[28] B. Canuel et al. Exploring gravity with the MIGA large scale atom interferometer. Science reports, 8:14064 (2018)

[29] Christos Merkatas et al. Shades of dark uncertainty and consensus value for the Newtonian constant of gravitation. arXiv:1905.09551v1 (2019)

[30] Li, Qing et al. Measurements of the gravitational constant using two independent methods. Nature 560, 582–588 (2018)

[31] P. J. Mohr, D. B. Newell, and B. N. Taylor, CODATA recommended values of the fundamental constants: Rev. Mod. Phys. 88, 035009 (2014)

[32] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. M. Tino. Precision measurement of the Newtonian gravitational constant using cold atoms, Nature 510 (7506), 518–521 (2014)

[33] S. Schlamminger and R.D. Newman. Recent measurements of the gravitational constant as a function of time. Phys. Rev. D 91, 121101 (2015)

[34] C. Rothleitner and S. Schlamminger. Measurements of the Newtonian constant of gravitation, G. Rev. Sci. Instrum. 88, 111101 (2017)

[35] G. Rosi, Challenging the big *G* measurement with atoms and light. J. Phys. B: At., Mol. Opt. Phys. 49(20), 202002 (2016)

[36] Rutherford E. The scattering of α and β particles by matter and the structure of the atom. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science. Series 6. 21 (125): (1911)

[37] Hofstadter R. et al. High-Energy Electron

Scattering and Nuclear Structure Determinations. Phys. Rev. 92, 978 (1953)
[38] T. Bayram, S. Akkoyun, S. O. Kara and A. Sinan, New Parameters for Nuclear Charge Radius Formulas, Acta Physica Polonica B. 44(8), 1791-1799 (2013)
[39] Max Planck. Ueber das Gesetz der Energieverteilung im Normalspectrum (PDF). Ann. Phys., 309 (3): 553–63 (1901)
[40] Niels Bohr. On the Constitution of Atoms and Molecules, Part I (PDF). Philosophical Magazine. 26 (151): 1–24 (1913)
[41] https://en.wikipedia.org/wiki/Atomic_radii_of_the_elements_(data_page)
[42] Anirban Biswas et al. Explaining the 3.5 keV X-ray Line in a Lμ − Lτ Extension of the Inert Doublet Model. JCAP 02, 002 (2018)
[43] Nico Cappelluti et al. Searching for the 3.5 keV Line in the Deep Fields with Chandra: The 10 Ms Observations. The Astrophysical Journal, 854:179 (12pp), (2018)
[44] Astrophysics and Space Science Library. 412, Very Massive Stars in the Local Universe. Editor Jorick S. Vink. Springer (2014)
[45] https://openstax.org/books/astronomy/pages/18-2-measuring-stellar-masses
[46] RW Pattie Jr et al. Measurement of the neutron lifetime using a magneto-gravitational trap and in situ detection. *Science* 11, Vol. 360, Issue 6389, pp. 627-632 (2018)
[47] F. E. Wietfeldt et al. A path to a 0.1 s neutron lifetime measurement using the beam method. Physics Procedia 51, 54 – 58 (2014)
[48] Seshavatharam UVS and Lakshminarayana S. Neutron life time enigma in the light of Electromagnetic and Weak gravitational constants. Nucleus-2019, Dubna, Russia, Book of abstracts, Page 242 (2019)
[49] C.T. Russell and M.K. Dougherty. Magnetic Fields of the Outer Planets. Space Sci Rev, 152: 251–269 (2010)
[50] Humphreys, D. R. The Creation of Planetary Magnetic Fields. Creation Research Society Quarterly 21(3), (1984)
[51] Joseph Polchinski. String theory to the rescue. arXiv:1512.02477v5 [hep-th] 16 Dec 2015
[52] Ralph A. Alpher, George Gamow and Robert Herman.Thermal cosmic radiation and the formation of protogalaxies. PNAS, 58 (6) 2179-2186. (1967)
[53] F. Englert and R. Brout. Broken Symmetry and the Mass of Gauge Vector Mesons. Physical Review Letters, vol. 13, Issue 9, pp. 321-323 (1964)
[54] Higgs P. Broken Symmetries and the Masses of Gauge Bosons. Physical Review Letters. 13 (16): 508–509 (1964)
[55] The ATLAS Collaboration. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. Phys.Lett. B716, 1-29 (2012)