

### **SIMULATION OF A DETERMINISTIC MODEL OF HIV TRANSMISSION BETWEEN TWO CLOSED PATCHES**

#### **ABSTRACT**

In this paper, Numerical simulation of a deterministic model of HIV transmission between major cities in Kenya is carried out. The model considered two closed patches connected by the commuter movements of truck drivers being the agents of HIV transmission. The transmission kernel being the function of distance between the patches is ignored. The numerical algorithms are applied in the solution of a nonlinear first-order differential equations. The algorithms are implemented with the aid of MATLAB solver which has an in-built mechanism of RungeKutta method of fourth order. Numerical simulation indicated the population dynamics of the patches, effect of migration on female sex workers and model reproduction number. The findings of the study were that the migration of the truck drivers between two closed patches contributed significantly to the spread of HIV. In this regard, it was recommended that stakeholders should target the truck driving population and towns along the transport corridors to mitigate the growing HIV infections and integrate the truck drivers in the national health strategy.

**KEY WORDS:** Basic reproduction number, HIV/AIDS, MATLAB, Migration, Monod function, Runge Kutta

#### **1. Introduction**

Simulation is the application of the developed model to the problem or question and then leads to the derivation of a strategy to solve the problem or answer the question, Velten, K. [16]. A numerical simulation is a design that is run on a computer following a program that implements a mathematical model for a physical system. Numerical simulations are required to study the behaviour of systems whose mathematical models are too complex to provide analytical solutions, as in most nonlinear systems. Numerical methods are used to obtain approximate numerical solutions to the ordinary differential equations derived from the models. Mathematical modeling and simulation are important research and monitoring tools used to understand biological communities and their relationships to the environment. Mathematical models are collections of variables, equations and starting values that form a cohesive representation of a process or behavior. The basic reproduction number is a key parameter, which determines the behavior of the model. In epidemiology, it is essential to quantify the severity of actual or potential outbreaks of infectious diseases. The reproduction ratio is defined as the average number of secondary infections produced when one infected individual is introduced into a host population where everyone is susceptible.

#### **2.0. Related literature**

Mirgichan et al [7] developed a deterministic model of HIV transmission between two closed patches incorporating the Monod equation in migration. The model was developed using the population based compartmental structure and adopted SI metapopulation model and the population of each patch was divided into three sub-populations namely susceptible individuals (S), the infected individuals (I) and individuals who developed AIDS (A). The two patches were connected with a migration parameter,  $m$  following the Monod function. The transmission kernel  $K$  which is a function of distance between the

patches was ignored. The long-distance truck drivers were considered as the agent of HIV transmission between the patches. The transmission of HIV infection was assumed through sexual contact between the truck drivers and their clients female sex workers. Six ordinary nonlinear first order differential equations were deduced from the model structure. The model was evaluated and analyzed using differential and integral calculus and also computational mathematics-Wolfram Mathematica software. The Monod function was used to estimate the effect of the number of sex workers on migrations. The basic reproduction number was determined using the Next Generation Matrix method. The HIV infection free equilibrium point was determined and its local and global stability investigated. This paper continues the previous study by Mirgichan et.al [7] to obtain numerical solutions to these equations and show the dynamics of the state variables within a given time range. The effect of migration parameter on the number of female sex workers in the patches is assessed numerically. At the same time the effect of basic reproduction number on migration parameter is analyzed. Sohaib, A. & Muhammad [10] implemented two numerical schemes namely Galerkin-Petrov and Legendre wavelet collocation Method for the approximation solution of the mathematical model which described the behavior of CD4+ T-cells, infected CD4+ T-cells and free HIV virus particles after infection. The study discussed and analyzed the effect of constant term and different variable source terms used for the supply of new CD4+ T-cells for the dynamics of CD4+ T-cells, infected CD4+ T-cells and free HIV virus. The model was also solved using fourth order Runge Kutta method. The validity and reliability of the proposed schemes were verified by comparing the numerical and graphical results with the results of RK4-method. Comparison of the numerical and graphical results of cGP(2) and LWCM with RK4-Method confirmed that cGP(2) and LWCM performed excellent accuracy. The study highlighted the accuracy and efficiency of the proposed schemes with the other traditional schemes.

### 3.0. Description of model variables and parameters

The set of parameters and variables used in the proposed model are described in Table 1 and 2

Table 1. Model Variables

Variables	Description
$S_1(t)$	The number of susceptible individuals in patch A at time t
$I_1(t)$	The number of HIV infectives in patch A at time t
$A_1(t)$	The number of Individuals who have developed AIDS in patch A at time t
$S_2(t)$	The number of susceptible individuals in patch B at time t
$I_2(t)$	The number of HIV infectives in patch B at time t
$A_2(t)$	The number of Individuals who have developed AIDS in patch B at time t

Table 1. Model parameters

Parameters	Description
$\beta_1$	HIV Infection rate for patch A
$\beta_2$	HIV infection rate for patch B
$\lambda$	Transmission rate/force of infection
$\gamma$	the Progression rate of HIV infected individuals to AIDS class
$\delta$	the induced -death rate due to AIDS
$m$	the specific rate of migration of individuals
$Z$	the concentration of resource availability for migration
$k_z$	the half-velocity constant
$m_{max}$	The maximum specific migration
$N(t)$	Total population of the patches with time

#### 4.0. Basic mathematical equation for the proposed model

$$\left. \begin{aligned} \frac{dS_1}{dt} &= -\lambda_1 S_1 + m_{max} \left( \frac{z}{K_z+z} \right) (S_2 - S_1) \\ \frac{dI_1}{dt} &= \lambda_1 S_1 - \gamma I_1 + m_{max} \left( \frac{z}{K_z+z} \right) (I_2 - I_1) \\ \frac{dA_1}{dt} &= \gamma I_A - \delta A_1 + m_{max} \left( \frac{z}{K_z+z} \right) (A_2 - A_1) \\ \frac{dS_2}{dt} &= -\lambda_2 S_2 + m_{max} \left( \frac{z}{K_z+z} \right) (S_1 - S_2) \\ \frac{dI_2}{dt} &= \lambda_2 S_2 - \gamma I_2 + m_{max} \left( \frac{z}{K_z+z} \right) (I_1 - I_2) \\ \frac{dA_2}{dt} &= \gamma I_2 + m_{max} \left( \frac{z}{K_z+z} \right) (A_1 - A_2) - \delta A_2 \end{aligned} \right\} (1)$$

#### 5.0. Methodology

In this sub-section, we describe the Runge Kutta method and how it is manifested in the model equations and there after we carry out numerical simulations by fourth order Runge Kutta method in MatlabR2017a to study the dynamical behavior of the model state variables in the presence of model parameters. The Runge Kutta method is a numerical method of solving an initial-value problem of an ordinary differential equations. Given the initial conditions and the step size , the method involves finding the  $k$ -values for a particular order and finally take the values of  $k$  and substitute them into the Runge Kutta formula to obtain the numerical solution. The preceding value of  $k$  is used to get the subsequent values of  $k$  using the same interval. Solving an ordinary differential equation using the Runge Kutta method manually becomes strenuous. At the same time, choosing the best step size to obtain good results is also difficult, Epperson, F.J. [3]. In our case, the developed HIV deterministic model is evaluated using an ordinary differential equation which is an initial value problem. We therefore use an inbuilt Runge Kutta method of order 4 in MATLAB which also uses the best step-size to obtain better numerical results. The model equations are coded in MATLAB , initial conditions assigned and the numerical simulation is run automatically to yield numerical result. We considered the migration of truck drivers between two major cities in Kenya; Nairobi and Nakuru. In the model  $N_1$  and  $N_2$  represents the total population for Nakuru and Nairobi respectively, whose data was obtained from Kenya National Bureau of Stastitics,population projections of 2015 as tabulated in Table 1. According to Business daily [2], a minimum of 2000 truck drivers enters and leaves Nakuru-Nairobi daily. Model variables and parameter values are obtained from the available literature and HIV data.A summary of the model variables, parameters and their respective values and sources are shown in Table 3.

Table 3. Initial Conditions

Parameter	Value	Source	Initial conditions	Value	Source
$\beta_1$	0.02148/year	Kenya HIV county profiles 2016	$N_1$	1,959,880	KNBS population projection [5]
$\beta_2$	0.04224/year	Kenya HIV county profiles 2016	$N_2$	4,232,087	KNBS population projection [5]
$\gamma$	0.125/year	Baryarama F, Mugisha JYT, Luboobi LS., (2006)	$S_1(0)$	1,918,663	KNBS population projection [5]
$m$	0.000323/day	(Business daily, 2019)	$S_2(0)$	4,060,577	KNBS population projection [5]
$\delta$	0.01807	Kenya HIV county profiles 2016	$I_1(0)$	36065	Kenya HIV estimates ,2015
$I_2(0)$	150,071	Kenya HIV estimates 2015	$A_1(0)$	5152	Kenya HIV estimates, 2015
$A_2(0)$	21,439	Kenya HIV estimates (2015)			

## 6.0. Numerical results and discussion

Numerical simulations are obtained using the secondary data in table 3 using MATLAB R2017a and presented graphically as shown in Figure (1) to (14). The behaviour of the curves are explained.

### 6.1. Population dynamics of state variables

The numerical simulations in the figures below display the population size for Nakuru and Nairobi in Kenya within a given time range.

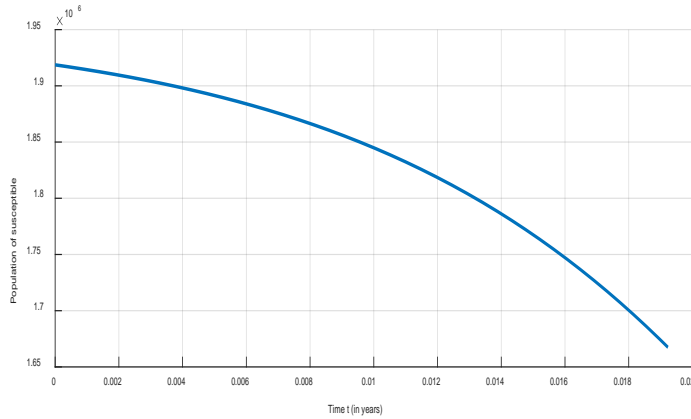


Figure 1. Population of susceptible individuals in Nakuru with time

The population of susceptible individuals in Nakuru decreases with time. This is because the susceptible individuals get infected with time and the infected individuals move to the next class of infectious individuals since there are no births. At the same time a proportion of the susceptible individuals move to Nairobi thus the decrease.

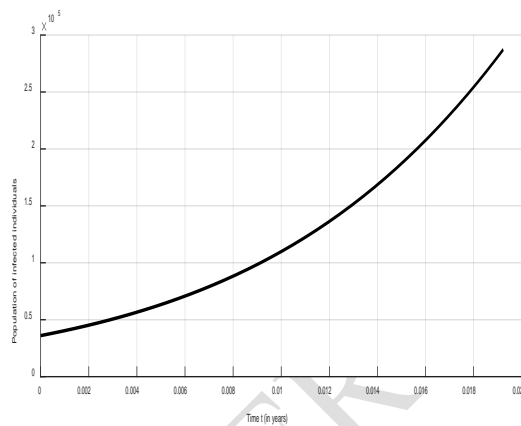


Figure 2: Population of infected individuals in Nakuru with time

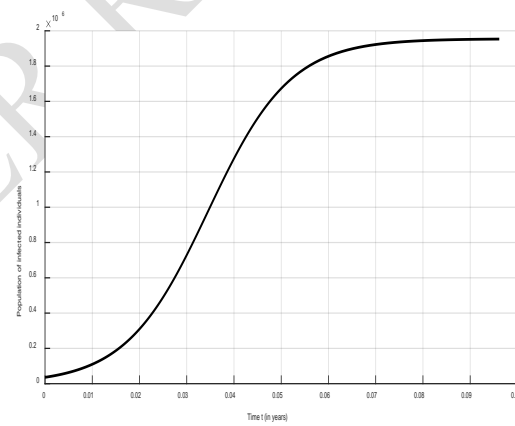


Figure 3. Population of infected individuals in Nakuru as time increases

Initially, the population of infected individuals in Nakuru increases with time. However, as time converges to infinity at  $t = 0.08$  years, the population of infected individuals remains constant after attaining a maximum value. The increase of this population is as a result of the addition of infected individuals who have moved from Nairobi to Nakuru. At the same time, the susceptible individuals in Nakuru become infectious. For large values of  $t$ , the curve obeys the Monod curve.

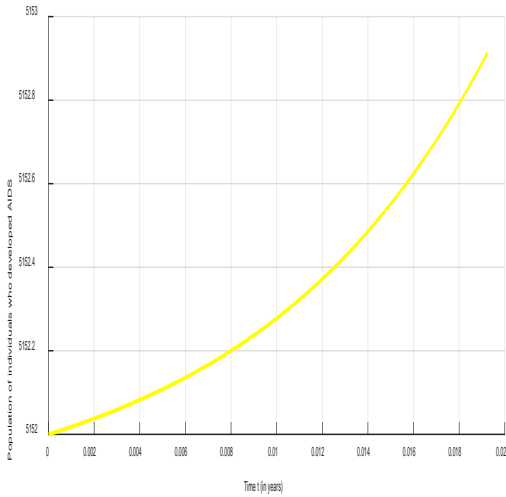


Figure 4. Population of AIDS individuals in Nakuru with time

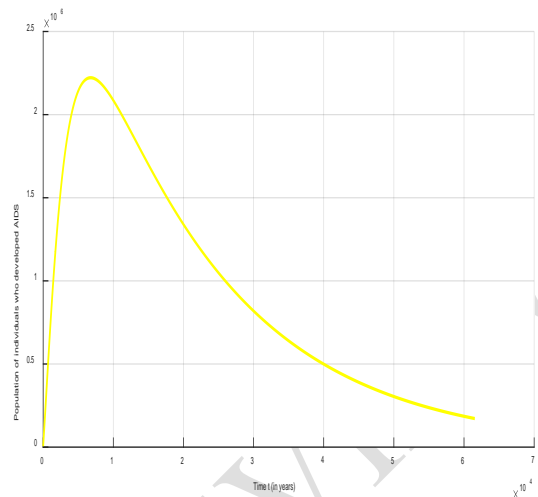


Figure 5. Population of AIDS individuals in Nakuru as time approaches to infinity

At the start, the population of individuals who developed AIDS increases with time. This is due to progression of the infected individuals to AIDS class and addition of individuals who developed AIDS from Nairobi. At  $t = 0.018$  years, the population diminishes slowly towards zero.

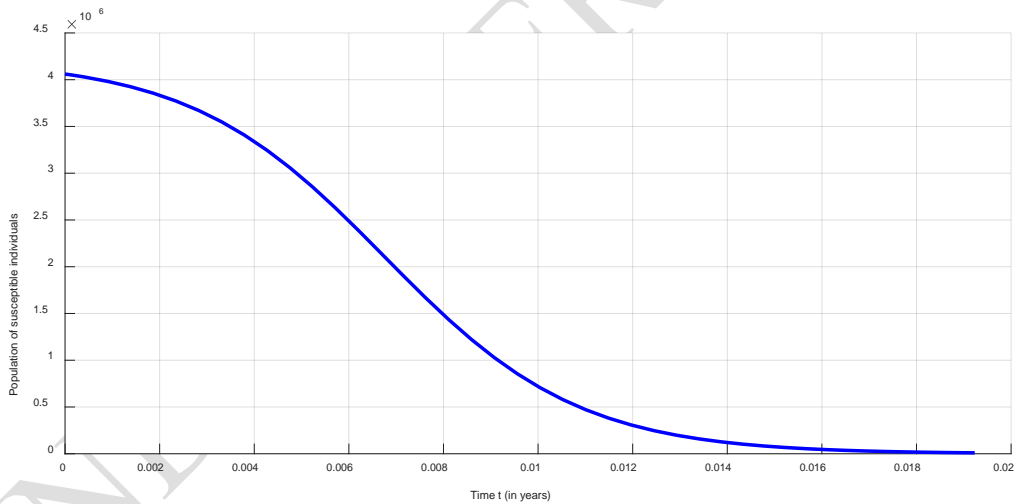


Figure 6. Population of susceptible individuals in Nairobi with Time

The population of susceptible individuals in Nairobi decreases with time and at  $t = 0.018$  years the population decreases to zero. This is because there are no births as well as transition of infected individuals to the next compartment at the same time migration of susceptible individuals to Nakuru.

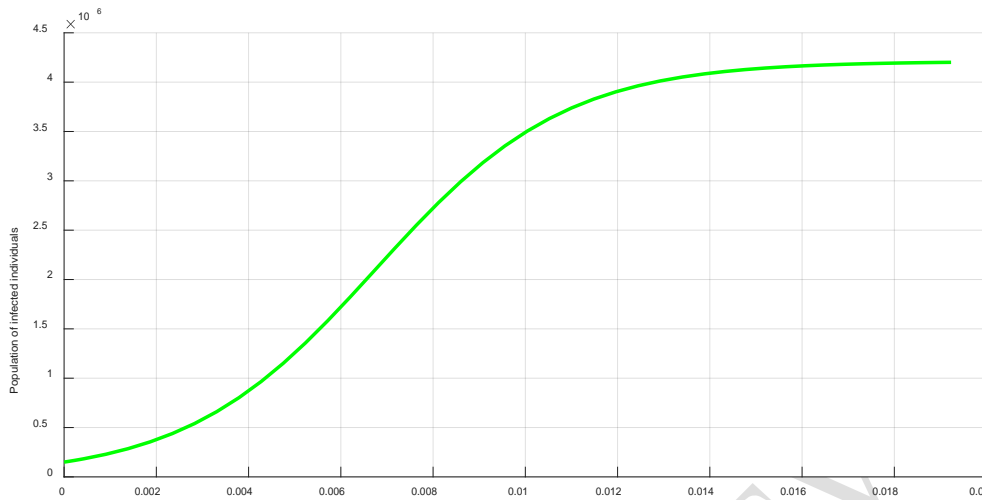


Figure7: Population of infected individuals in Nairobi

As susceptible individuals become infected, the population of infected individuals in Nairobi increases with time. A proportion of infected individuals from Nakuru also attributes to this increase. However, at  $t = 0.016$  years, the population reaches a maximum point and then remains constant throughout the infection period.

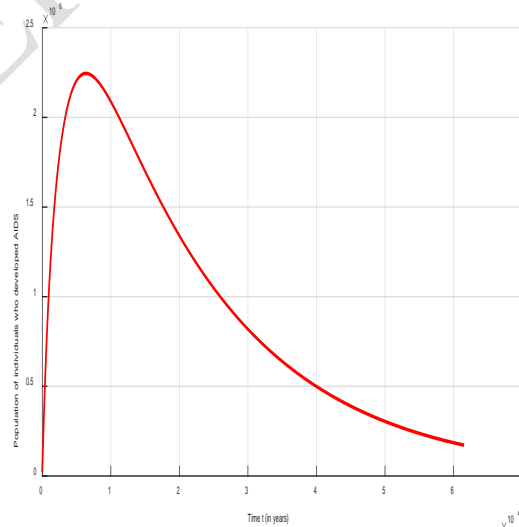
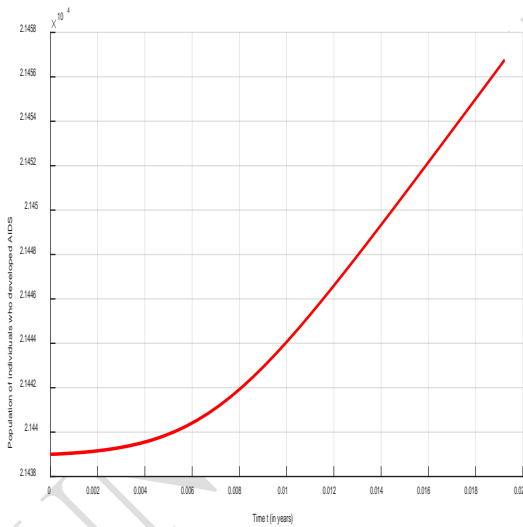


Figure 8. Population of individuals who developed AIDS in Nairobi with time Figure 9. Population of individuals in Nairobi as time approaches infinity

Initially, the population of individuals who developed AIDS in Nairobi increases gradually for 0.008 years and afterwards increases rapidly with time. This is due to the infected individuals progressing to AIDS class and addition of individuals who developed AIDS from Nakuru. However, as time approaches infinity, at first the population increases rapidly before it reaches a maximum value at 5000 years and then decreases slowly to zero with time.



## 6.2. Effect of Migration parameter, m on Female Sex Workers, Z

The migration parameter follows the Monod function,  $m = m_{\max} \left( \frac{Z}{k_z + Z} \right)$  where  $K_z = 0.25$  obtained from the experimental data for bacterial growth by Smith, H. [9]. In our model we use  $z$  as the Female Sex Workers (FSW). From table 3, the migration rate is 0.000323/day, thus the modified Monod equation becomes,  $m = 0.000323 \left( \frac{Z}{(0.25 + Z)} \right)$ . According to Daily Nation, 1<sup>st</sup> April 2014, there are about 200 female sex workers in Nakuru and about 27,620 female sex workers in Nairobi. The maximum value may be obtained for a period of three weeks or a month.

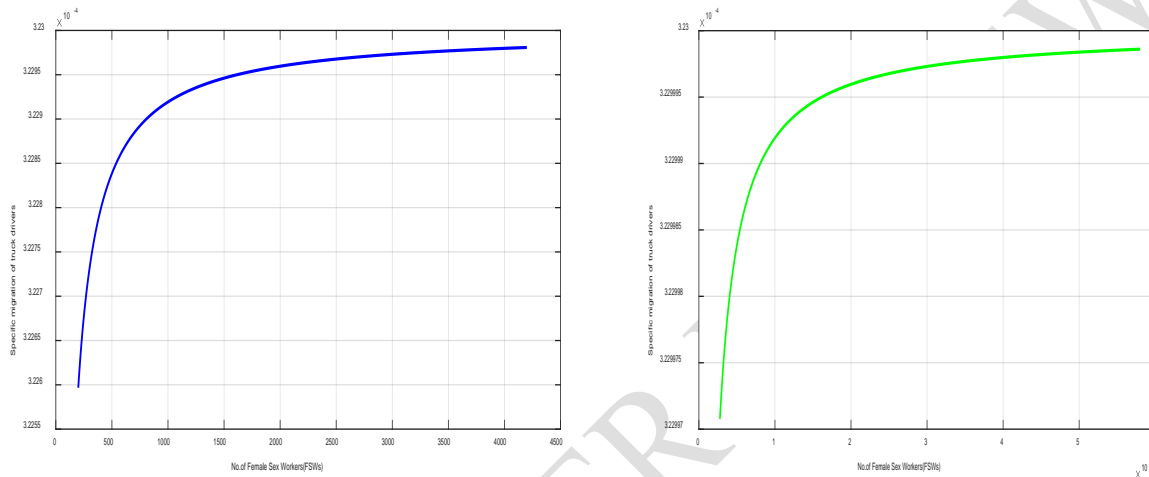


Figure 10. Migration of truck drivers and Figure 11. Migration of truck drivers and Female Sex Workers in Nakuru Female sex workers in Nairobi

From the above curves, the rate of migration increases slowly with increase in the number of female sex workers in Nakuru and rapidly increases with increase in the number of FSW, until a maximum value is reached for both towns. However, the saturation point is realized much earlier in Nairobi than in Nakuru.

### 6.3. Effect of migration parameter, m on the basic reproduction numbers

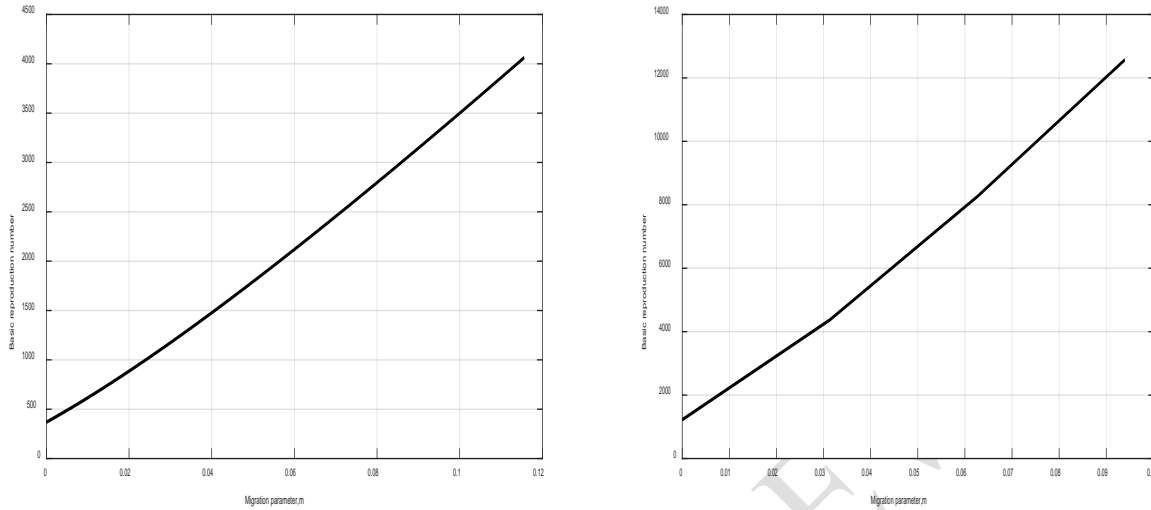


Figure 12. Basic reproduction and migration for Nakuru Figure 13. Basic reproduction and migration for Nairobi

The basic reproduction number increases with increase in the migration parameter for both cases. But for the same migration parameter, the basic reproduction number for Nairobi is higher than that of Nakuru.

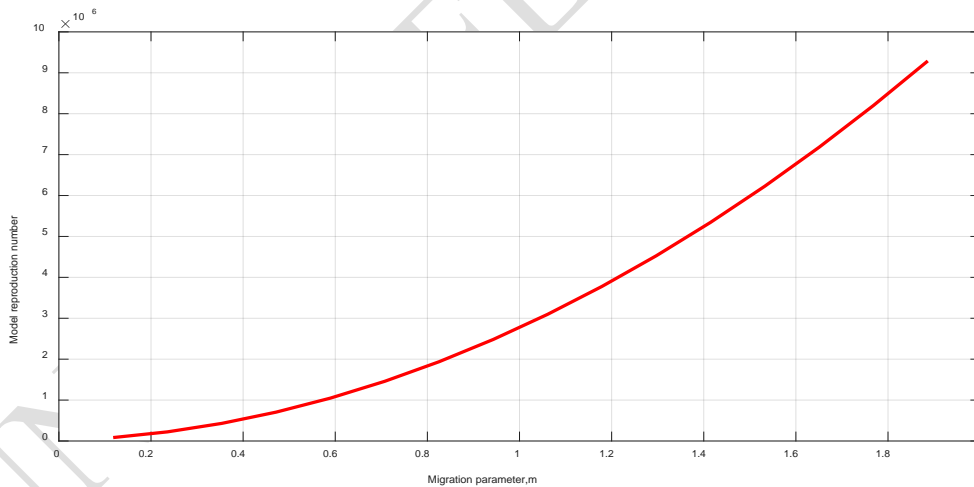


Figure 14: Migration parameter and model reproduction number for Nakuru and Nairobi

The model reproduction number grows from 0.1 exponentially with increase in the migration parameter. This trend is attributed by interactions between the truck drivers and their clients, Female sex workers. The truck drivers establish multiple sexual partners and relationships in the cities, thus increasing the model reproduction number.

## 7.0. Conclusion

The graphical results of the simulations of the deterministic model of HIV transmission between two closed patches shows that for a closed system, that excludes birth and natural death rates, the population size of the susceptible individuals' decay exponentially while the infected and individuals who developed AIDS grow exponentially with time. With the net migration, the infection rate, the progression rate and HIV induced death rate varies. An increase in the migration parameter leads to an increase in the basic reproduction number. This infers that the migratory nature of the long-distance truck drivers along the transport corridors contributes significantly to the spread of HIV. According to Monod growth theory, the number of truck drivers in a patch increases with increase in the number of female sex workers. The truck drivers are enticed by the sex workers due to delays caused by congestion and bureaucratic procedures at the ports or depots as well as inadequate parking spaces. The Monod curves showed that migration increases with increase in the model reproduction number and the number of female sex workers. Minimizing the number of truck drivers moving between the cities at a time reduces the chances of contracting HIV, since this will reduce congestion at the depots. From the research findings it is recommended that health practitioners, government policy makers and institutions of supply chain management should make deliberate efforts in regulating the migration of drivers to regions or transport routes used by truck drivers. The health implications of this observation are that keeping the reproduction number below unity may be necessary in reducing the spread of HIV infection by truck drivers. Health care facilities and services should also be provided along the transport corridors to mitigate HIV transmission. Creation of more collection depots reduces the chances of truck drivers engaging in a risky behavior. Further research work should focus on incorporating vital dynamics, increase the patch size as well as varying the migration parameter.

## 8.0. References

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