
Distribution of income (including minimum wage); comparison of societies as studied by a molecular-dynamics algorithm and analytical techniques

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Original Research Article

Abstract

In this research we tried to answer the question: How to optimize the total production of economy. For finding the answer we used two postulates: First, a worker is incentivated to work if it pays off. When in the income ranking the neighbor below earns less and the one above earns more the worker will work harder and produce more. The productivity of the worker is proportional to this 'derivative' in the income curve. (Note: a worker's salary is not assumed necessarily proportional to his productivity). The second postulate depends on who is in control of the production process. In highly-simplified naming: In capitalism, the capital takes the decisions, in a democracy the people do, by vote. We also simulated a dictatorial system in which decisions can be imposed by a (benevolent) dictator.

We used these ingredients in evolutionary computation. Starting with an arbitrary initial distribution, we make random small changes to it and if the total production increases, a decision will be made whether to implement these changes. This procedure is repeated until the distribution is stable. Remarkably, the outcomes for 'democracy' and 'capitalism' are similar. Capitalism and democracy go hand-in-hand together: One person getting all income, two people working, and most not working and not receiving. These results are also analytically found.

In 'communism', nobody works and everybody perishes. In a 'dictatorial' system we can optimize production for the benefit of the people, and come to the conclusion that the introduction of minimum wages is beneficial, and these should be 50% of the average income.

Keywords: Income distribution, Minimum wage, Molecular dynamics simulation, Macro-economy

1 Introduction

One of the prime goals of a society is to organize economy in such a way that it optimizes the production of goods, assuming that wealth is equal to the consumption of goods and consumption

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makes people happy. This link between wealth and happiness can be questioned but seems logical[22] and a correlation has been found between the two[25]. In fact, it is the central tenet of the philosophical school of Utilitarianism of Jeremy Bentham (1748-1832) and John Stuart Mill (1806-1873) who aim for "The greatest good for the greatest number"[19]. The question is then how to reach this goal? If we want to maximize this, we first have to quantify it and then maximize the value of the quantifier through some mechanism. Quantization of the real world is a cumbersome, painstaking and often fallible process but large international institutes, like the OECD, are trying to do this, for instance the wealth (capital stock) of society[14] and even the subjective concept of well-being[15]. Considering the difficulty, we might thus, alternatively, try to simulate virtual worlds – where things are easier to measure – and find out how to optimize production in these virtual worlds instead. This is what we endeavored in this research. We stripped the model from any distracting decorations to uncover the bare core of the ideas. In the process we use archetypal names for things.

In a first approach, we slightly changed this goal and tried to answer the question: How to optimize the *total* production and, moreover, assume Say's Law that "Every supply creates its own demand", thus, all the produced goods will be consumed and add to the welfare of the people. To find an answer we used the following postulates. Either can be questioned, but seem quite reasonable. The first one is the prime tenet of this work:

- 1 A worker will be incentivated to work if it pays off (even when we know money does not make a person happy[13]). When in the income ranking the neighbor below earns less and the one above earns more the worker will work harder and produce more. We assume the productivity of the worker is proportional to this 'derivative' in the income curve. The output and wealth of a nation is thus directly linked to the workers' labor ("the exchange value of a commodity [is] reducible to the amount of labor embodied in it"[26]).

To add to this, we will make assumptions about the production process itself. In a so-called 'capitalist' system[1, 5], the axiom goes as follows:

- 2a The capital, by definition in control of the decisions in production, will want to maximize total production since the profit is empirically always about 5% of production[18]; increasing production will increase profit for the capital and capital will thus decide to do so and also manage to implement this in the free market of capitalism.

(Note: a salary is not assumed necessarily proportional to the productivity of a worker). Of course, there can be other ways of organizing society and economy. An alternative can be:

- 2b In a democracy, a worker will vote for changes to the income distribution. A reasonable worker will vote in favor of the distribution if it gives the worker more *wealth* (not necessarily more income, as we will see; it is not relevant how much euros are earned, but how much can be bought for them).

Note that this latter axiom does not necessarily optimize total production, but only aims for (marginal) approval rate; the capitalist axiom is abandoned if workers take the decisions in the production process.

In what follows, the comments about the results obtained are always within the framework of this virtual created world and not necessarily pertain to the real world. If we, for instance, comment on the results of the 'capitalist' simulation, it is about that simulation, and not about the real capitalist world, although the readers can determine for themselves how much of it is relevant for the real world.

2 Results and discussion

We used these postulates in evolutionary computations, or 'molecular dynamics', that works in the following way. Starting with an arbitrary initial distribution, we make random small changes to it and if the total production increases (if the 'energy' is lower), the capital will decide to implement these

changes; in the molecular dynamics simulation we keep them. This procedure is repeated until the distribution is stable. We call this the final state.

The distribution of people's income is given in the sorted vector p , where an element p_i represents the income in euros of person i , with i running from 1 to N , the total number of people simulated. We call this the 'percentile' (which is also adequate when the number of persons simulated is 100). The production vector w is given by the 'derivative' of the income curve, see Figure 1,

$$w_i = \frac{p_{i+1} - p_{i-1}}{2}, \quad (2.1)$$

with the two special boundary cases given by

$$\begin{aligned} w_1 &= \frac{p_2 - p_1}{2} \\ w_N &= \frac{p_N - p_{N-1}}{2}. \end{aligned} \quad (2.2)$$

The unit of production is joule. In these equations we thus intrinsically assume a proportionality factor α of one joule of production for every euro in the income derivative, $\alpha = 1$ joule/euro. Since we are only interested in the relative effects of distribution, this is allowed and facilitates our calculations; the factor α is omitted from the equations but should be imagined there, at least to make the units correct (joules on both sides). The total income P and production W are given by, respectively,

$$\begin{aligned} P &= \sum_{i=1}^N p_i, \\ W &= \sum_{i=1}^N w_i. \end{aligned} \quad (2.3)$$

It is not so relevant how much a worker gets paid in euros. More interesting to know is what a worker can *buy* for the income, since that adds to wealth. That is also one of the reasons we keep the total income fixed at a value equal to the number of workers N , so for 100 workers, $P = 100$ euro; the money itself is just a bookkeeping number to keep track who has right to what share of the total consumption, and there is no loss of generality if we keep this value fixed. An average worker thus always has $P/N = 1$ euro of income. With W joules being produced, a euro can buy W/P joules of goods, which is the average salary of a worker Y . A specific individual worker i thus has an income p_i euros and a 'real income' (wealth) in terms of joules buying power equal to

$$c_i = \frac{p_i W}{P} (\text{J}). \quad (2.4)$$

Finally, we can also define an average price of things, $Z = P/W$, so we can also define the wealth of worker i as $w_i = p_i/Z$, the income divided by the average price of things. Now that we have defined all the necessary concepts, we can start making simulations.

We can start with a full-equality distribution; for the sake of a better name and with a sense of humor inspired by Orwell's *Animal Farm*, we call it 'communism'. (In communism people get paid by their needs and there is thus no financial incentive to work, the thing we simulate here; people ideally work according to their capabilities. "From each according to his ability, to each according to his needs"[11], probably not giving full justice to marxism[24]). In this system we have a set of $N = 100$ people that all earn equally ($\forall i : p_i = 1$ unit). The total income is $P = 100$ according to Eq. 2.3. In the absence of incentives (with all p_i being 1, all w_i are 0, according to Eq. 2.1), nobody is doing anything, total production is zero ($W = 0$) and people live in misery. The average price of things is infinite: $Z = P/W = 100/0 = \infty$. A worker, on average, has a real income of $1 \times W/P = 0$. A modal worker (percentile 50) also has a real income equal to his share of income: $c_{50} = p_{50} \times W/P = 0$. In fact, any worker has a real income of 0. (See Table 1 for a summary of important results). Society incentivating laziness and dying. "Why should I work? No benefit to be gained from it!"

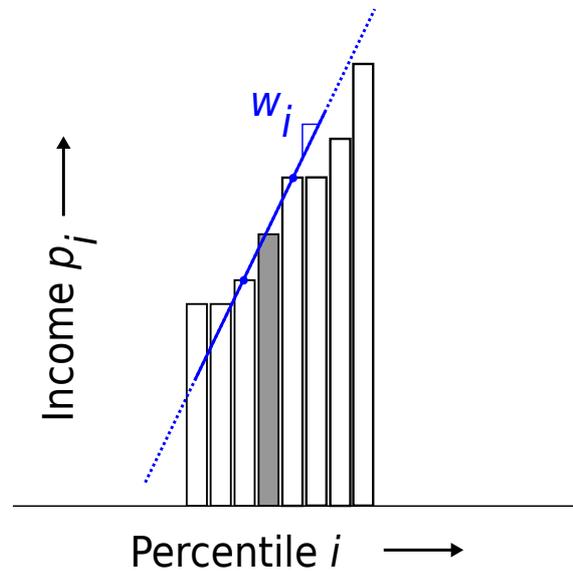


Figure 1: Detail of the distribution of income p_i as a function of percentile i . The production of worker i (shaded) is given by the 'derivative' of the income curve, namely the difference (divided by two) of the incomes of the two immediate neighbors (shown by the blue line), Eq. 2.1.

We can now make a small change to the distribution. We take 1% away from the income of any worker and give it to any other worker. After that we'll sort the array, placing the lowest salary in percentile 1, $p_1 = 0.99$, and the highest salary in percentile 100, $p_{100} = 1.01$. All the others remain unaltered, $p_i = 1$ for $i = 2 \dots 99$. See Figure 2. We now see that not only the one with higher salary ($i = 100$) starts working, but also worker 1 (who had his salary reduced), as well as workers 2 and 99:

$$\begin{aligned}
 w_1 &= (p_2 - p_1)/2 = (1 - 0.99)/2 = 0.005 \\
 w_2 &= (p_3 - p_1)/2 = (1 - 0.99)/2 = 0.005 \\
 w_{99} &= (p_{100} - p_{98})/2 = (1.01 - 1)/2 = 0.005 \\
 w_{100} &= (p_{100} - p_{99})/2 = 1.01 - 1 = 0.005.
 \end{aligned} \tag{2.5}$$

All others remain inactive. The total production is now given by the sum of the above numbers, $W = 0.02$. The total income remains unaltered at $P = 100$ euro (since we just transferred salary from one person to another). The price of things has dropped, from infinity to $Z = P/W = 100/0.02 = 5000$ euros/J. A worker, on average, has a real income (in terms of goods that can be bought) of $1 \times W/P = 0.2$ mJ; a few crumbs. Even people that do nothing ($i = 3 \dots 98$) get some crumbs, $c_{50} = p_{50} \times W/P = 1 \times 0.02/100 = 0.2$ mJ. The poorest worker gets a little less (99% of that) and the richest a little more (101% of c_{50}). Note that, while we took away income from someone, *everybody* benefits, even that person that was fleeced! And even the 'parasites' – the majority of the population – that do nothing. Moreover, in a system where capital works on a for-profit-basis, and profit is proportional to production – Piketty demonstrates an empirical rule of 5% per year – and the capital is the entity that makes the decisions in production in a free market – a.k.a. 'capitalism' – the

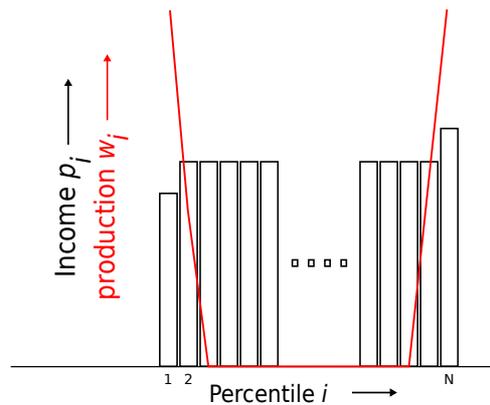


Figure 2: Distribution of income p_i and production w_i as a function of percentile i , after a single step in income redistribution, as explained in the text.

system will decide to implement these changes somehow.

This is not a work on promoting a political dogma, pitting capitalism against communism[2], nor will we discuss *how* capitalism manages to implement the changes (for instance through a government that tries to optimize this GDP), we just assume that these changes are implemented, with the justification for the assumption given in the introduction. Encouraged by the above results, we can see how the system will evolve. We now used the following procedure: We take a random person and transferred a random part of his income to a neighbor. We then sort the income vector p and calculate the production vector of the workers w and the total production scalar W . If this production increased, the changes were kept. If not, the previous distribution was kept. Then a new iteration was made. These steps were repeated until the distribution did not change significantly anymore. The algorithm was implemented in a Pascal imperative programming language that is publicly available in the Open Source domain, namely Free Pascal[3] that is compatible with Turbo Pascal. It was used as the main compiler in the IDE (integrated development environment) called Lazarus 1.6[9], running in a 64-bit operating system Linux Mint 18 (Sarah)[10]. For the figures a special graphics toolbox – EPSTool – was written by the author in Pascal that outputs in Encapsulated Postscript (EPS)[28] files that were linked directly in the LaTeX[8] source code of this document. The Pascal source code of the simulations implementing the algorithms of this document is available upon request.

The final distribution for this capitalist system is given in Figure 3. As we can see, it goes completely off-scale. It seems nearly all workers have zero income and zero production, while a small group at the top do all the work and get all the income.

First of all, the total production has gone up to $W = 100.0$ J, with the total income at $P = 100$ euro, the average worker has $1 \times W/P = 1.0$ joule of real income; prices dropped to $Z = 1.0$ euro/J. All workers $1 \dots N-2$ get no income whatsoever (it is within the range of significance of the calculations), $p_1 \approx 0$, while they also produce nothing. This includes the modal worker. His real income is $c_{50} \approx 0$. The lion's share of consumption goes to the top percentile, who has an income of $p_{100} \approx 100$ euro for which he can buy basically all 100 joules of production. This guy – 'top management' – also has to work a lot, $w_{100} \approx 50$ J, but is also flanked by 'middle management' that works a lot, $w_{99} \approx 50$ J, but who has no income $p_{99} \approx 0$ euro, like the modal worker.

Capitalism seems to be a real society wrecker. Most people get crumbs, if they get anything at all, while a few people get most, if not all, of the wealth. It is worse than the Iron Law of Wages of Lassalle, who proposed that real wages always tend, in the long run, toward the minimum wage

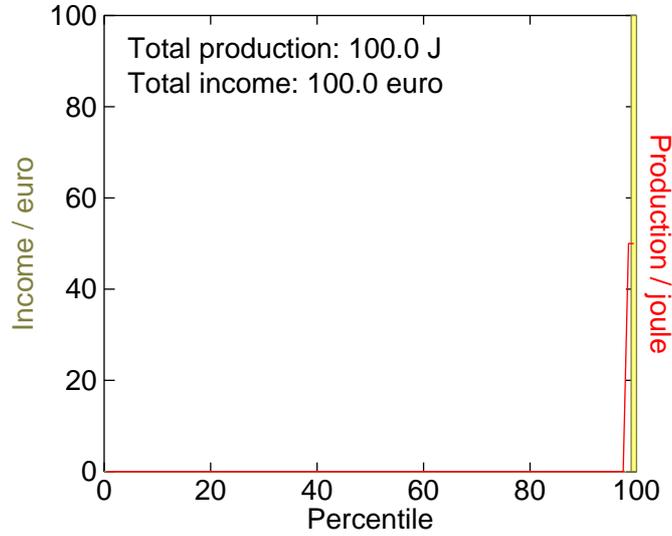


Figure 3: Final distribution of income p_i (yellow rectangles) and production w_i (red line) as a function of percentile i in a capitalist system.

necessary to sustain the life of the worker[7]. In our simulations, the wages sink below that. The simulations are quite slow in converging, but the tendency is for this to emerge. The outcome is inevitable. We can actually analytically prove that this outcome is optimal. Especially easy in the continuous domain. We maximize the sum of differences between adjacent elements in a sorted (monotonically increasing) positive valued vector whose sum of elements is constant. Translated to the continuous domain: Any monotonically increasing positive function $f(x)$ with a constant integral (in our case $\int f(x)dx = N = 100$) for which we want to optimize the integral of derivatives,

$$\begin{aligned} f_{\max} &= \max \left(\int_{\mathbf{R}} f'(x)dx \right) \\ &= \max (f(R_{\text{high}}) - f(R_{\text{low}})), \end{aligned} \quad (2.6)$$

with \mathbf{R} the range (people) under consideration. The maximum function with a constant integral equal to N is the delta-dirac function

$$f_{\max} = \delta(x - x_0)N, \quad (2.7)$$

which goes to infinity at $x = x_0$. And because the function has to be monotonously increasing, x_0 has to be equal to the right edge of the defined interval, in our case $[0 \dots N]$, thus N or 100. In the discrete domain, the sum W of Eq. 2.3 is given by

$$\begin{aligned} W &= \frac{p_2 - p_1}{2} + \frac{p_3 - p_1}{2} + \frac{p_4 - p_2}{2} + \frac{p_5 - p_3}{2} + \dots + \frac{p_N - p_{N-2}}{2} + \frac{p_N - p_{N-1}}{2} \\ &= p_N - p_1 \end{aligned} \quad (2.8)$$

Maximizing this function thus means maximizing p_N and minimizing p_1 . The latter must be zero, because it cannot be negative. And with the sum of all values equal to N , the maximum value for p_N is N , leaving nothing for the others. We thus prove that the optimum capitalist distribution is a delta function: a single person receiving *all* the produced goods, and only two people working. It is just a

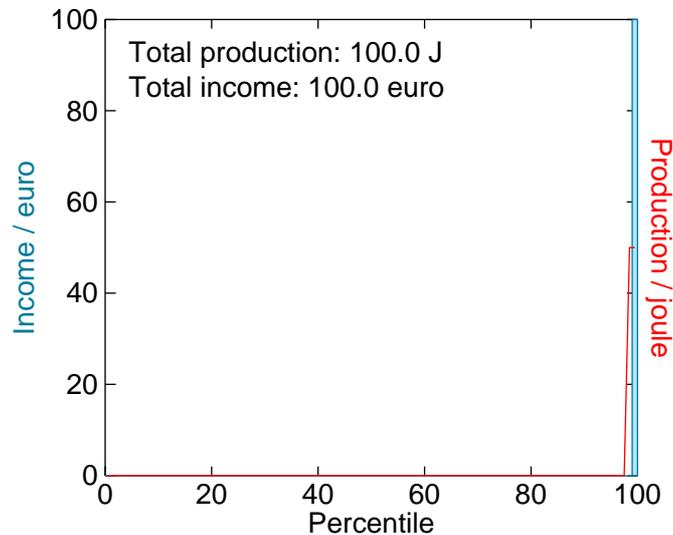


Figure 4: Final distribution of income p_i (blue rectangles) and production w_i (red line) as a function of percentile i in a democratic system.

matter of time (or sufficient iterations). Our numerical simulations corroborate this analytical result. In any case, it is better than the communist scenario where nothing is produced by anybody at all and everybody dies.

One might think here that taxation of the rich might solve the problem. However, this is naive, because if the distribution is leveled by taxation, this takes away the incentive to work harder as well. "Why would I work harder if the fruit of my labor will be taken away?!" What matters is how much money is taken home and into how many joules of goods it can be converted! That was the basic tenet of this work.

Capitalism simply optimizes production and does not care about the distribution, nor about 'fairness', which is a vague concept anyway. Still we see this economic system often as problematic and we try to see if we can actually come up with a solution. Democracy seems to be a good candidate. If the Utilitarian goal is the greatest good for the greatest *number* of people, we should have people *vote* for the distribution. This way, we can assure that at least the distribution is good for 51%, clearly better than the capitalist final solution (no pun intended).

The next simulations implemented this idea. Each worker remembers how much real income (in joules) he received in the previous iteration. Once again, a random change is made to the distribution, the production is calculated, and then the real income of each worker is determined. Now, this distribution is kept if more people vote in favor of it than against it. (Assuming people vote for a distribution that makes them wealthier). Figure 4 summarizes the results.

Contrary to our expectations, the distribution does not improve. In fact, it exactly mimics the capitalist case. The total production stays the same at 100 J and the distribution equal. Democracy is virtually indistinguishable from capitalism and this is in line with Milton Friedman's conjecture that capitalism is a necessary condition for political freedom[20], or we can say that freedom is a necessary condition for capitalism. Yet, how can it be that most people get absolutely nothing?! Even the first step after communism, presented in Fig. 2 gives more wealth to 99 of the 100 people. Why

Table 1: Summary of simulations for $N = 100$. p_i : income of worker i (unit: euro), c_i : wealth of worker i (unit: joule), P : total income ($= N \times 1$ euro = 100 euro), W : total production (unit: joule), Y : average consumption of workers ($= 1$ euro $\times W/P$; unit: joule), Z : average prices (unit: euro/joule).

Parameter	System			
	Communism	Capitalism	Democracy	Dictator (Minimum wage) $m = 0.5$
p_1	1.0	0	0	m 0.5
c_1	0	0	0	$m(1 - m)$ 0.25
p_2	1.0	0	0	m 0.5
p_{50}	1.0	0	0	m 0.5
c_{50}	0	0	0	$m(1 - m)$ 0.25
p_{99}	1.0	0	0	m 0.5
p_{100}	1.0	100	100	$100 - 99m$ 50.5
c_{100}	0	100	100	$(1 - m)(100 - 99m)$ 25.25
P	100	100	100	100 100
W	0	100	100	$100(1 - m)$ 50
Y	0	1.0	1.0	$(1 - m)$ 0.5
Z	∞	1.0	1.0	$1/(1 - m)$ 2

did they vote for the solution given in Fig. 4? The answer is that every tiny step brings more wealth to most people. Imagine we take away all the income from one person, say percentile 83 and give it to percentile 100. Because total production goes up, and most people do not see their income change, the wealth of most people goes up, and the new distribution is accepted with everybody's approval (well, one person, percentile 83 – now percentile 1 – couldn't care less, so he does not even bother to go to the ballot box, shouting "The system is rigged for the 1%!"). In the next step, all income (and wealth) from percentile 25 is taken away with a majority vote. And thus, successively, the wealth is taken away from all but one person.

Capitalism and democracy go hand-in-hand together, they have the same objectives. And this is in line with Milton Friedman's conjecture that capitalism is a necessary condition for political freedom[20], or we can even go so far as to say that freedom is a necessary condition for capitalism.

We could not even try to implement a system where people vote for an entire distribution, whatever it is (maybe the first step shown in Fig. 2, which is obviously better), because they will vote themselves for this distribution to be changed (as shown above). It is not even possible to cement it in a constitution, because easily a constitutionally necessary majority would vote for the constitution to be changed and go back to a system of 'vote for every step', since every step increases their wealth and a cry for 'liberty' will be heard in the streets, possible with an appeal to brotherhood and equality as well.

Let us now install an (enlightened) dictator that follows the school of early French economists, "trade, agriculture or manufacturing might make in augmenting the powers of the sovereign and rendering their subjects more prosperous"[26]. We can add features – whatever we can come up with – that are untouchable. And we try to help this dictator, who, we assume, wants to be good to

his people. Even so, it is not easy, because, as stated in the introduction, we first have to quantify the system. What parameter are we going to optimize? We can imagine a king that does not want to optimize the total production – which is easy and natural – but rather the 50% percentile, or something like this.

In the capitalist and democratic systems most people get zero income and they simply die. One thing is certain, and that is that a dead person does not produce and that is not in the interest of the people, nor of the capital. The famous phrase of Marx springs to mind, "What the Bourgeoisie therefore produces, above all, are its own grave diggers"[12], to which we can add, "... Bourgeoisie and Proletariat alike".

We can thus dictate a law that people should be kept alive. The way to do that would be to have a minimum wage. We will not enter here the discussion about the ideology or public attitudes of the concept of minimum wage[4], but just implement it at face value, the idea of having each worker receiving a minimum income and the impact on macroeconomics[21, 17]. And, ideally, this wage should not be in euros, but in joules (because you cannot eat money). This latter is tricky to do, because if we start with a system (like communism) that produces zero joules, we cannot start from the first iteration to demand a minimum energetic consumption for every worker, because there is no energy to distribute, yet. Thus, while not ideal, we put a minimum wage in euros anyway. In our simulation we start with an equal-income communist distribution, introduce a minimum income, and let it evolve naturally with this condition. Since democracy and capitalism and indistinguishable and capitalism is easier to simulate, we use the latter.

Using similar reasoning as used for establishing Eq. (2.7), the maximum discrete function with integral (sum) N and constant $C = m$ is one in which all-but-one person are getting m and one person (delta function) getting the rest, $N - (N - 1)m$, with m the minimum wage. For instance, for $m = 0.1$ and $N = 100$, everybody gets $m = 10$ cents, except one person who gets 90.1 euro instead. The total production is $W = (1 - m)N = 90.0$ joule. Prices are $Z = P/W = N/(1 - m)N = 100.0/90.0 = 1.11$ euro/J, average consumption is $Y = (1 \text{ euro})/Z = 0.9$ J. All workers get $c_i = m/Z = m(1 - m) = 0.09$ J consumption rights, except the 'boss', who gets $c_N = (1 - m)(N - [N - 1]m) = 81.09$ J. See Fig. 5. This value of W we also get when looking at Eq. 2.8: p_N is N minus the sum of all the other wages and maximizing p_N thus means minimizing the wages of all the others ($N - 1$ people): $p_1 = m$ and $p_N = N - (N - 1)m$, resulting in $W = p_N - p_1 = N(1 - m) = 90.0$ J.

We now have a way to optimize the wealth of the nation. We can optimize the wealth of everyone but one person, $N - 1$, by optimizing the functions $c_1 \dots c_{N-1} = m(1 - m)$ between the boundary values 0 (capitalism/democracy; no minimum wage) and 1 (communism; everybody income 1), see Figure 6. This maximum occurs for $m = 0.5$, that is, the minimum wage equal to 50% of the mean wage. The benevolent queen, from her royal point of view, sees that it is best for most people (for instance 99% if her tribe had 100 workers) and not even hurting so much the single one person who sees consumption reduced (from 100 J to 25.25 J). See Table 1.

Exactly this same distribution we get if we optimize the spendable income of percentile 50, c_{50} , without forcing a minimum wage. This, however, is much more difficult to implement; a political law can easily be implemented that requires payment of a minimum wage, but to impose a complete income distribution is more complicated if not unfeasible.

Interestingly, many countries have a minimum wage ratio that is close to this value. The Organisation of Economic Co-operation and Development (OECD) has data on some countries and most have a minimum wage in 2017 that is in the range 0.35-0.5 of the mean wages and 0.4-0.8 of the median wages[16]. (Our simulation gives 0.5 and 1.0 respectively). As an example, France has 0.50 and 0.62, respectively. We thus conclude that many governments are managing to implement a good income policy in their respective countries.

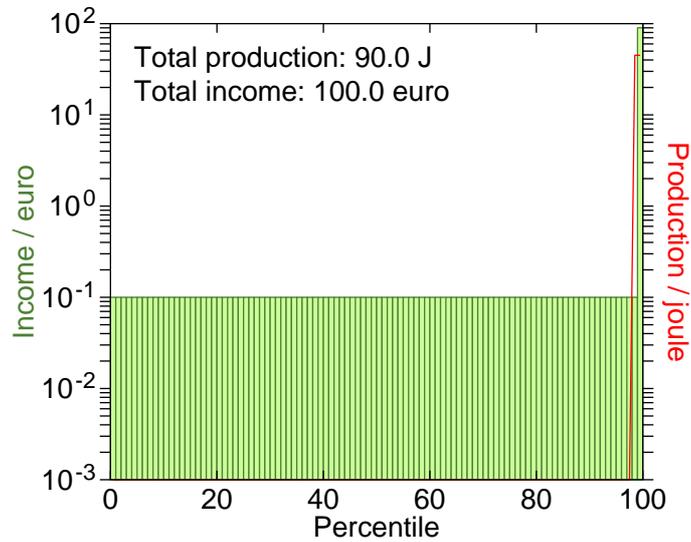


Figure 5: Final distribution of income p_i (green rectangles) and production w_i (red line) as a function of percentile i in a minimum-wage system. Logarithmic scale.

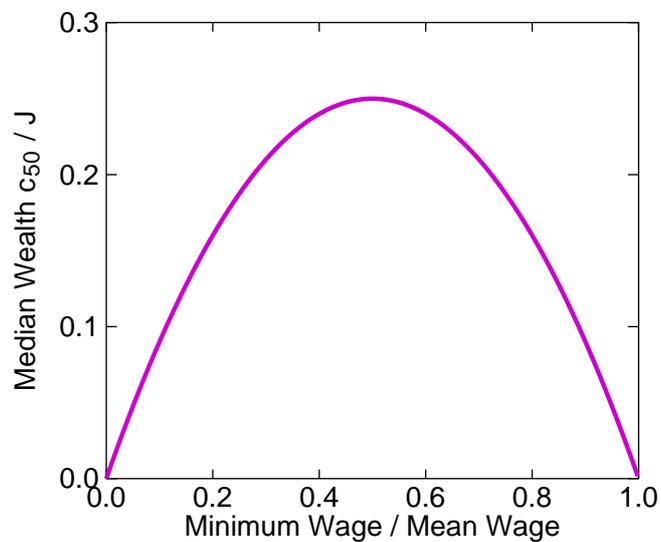


Figure 6: Wealth of the masses as a function of minimum wage. We see that capitalism and democracy (extreme left) and communism (extreme right) both lead to misery, but the introduction of a minimum wage equal to 50% of the mean wage optimizes the wealth of the masses.

3 Conclusions

The results of our computations can be summarized as:

- In a so-called 'communist' system nobody works and everybody dies; everybody gets an income, but you cannot eat money. This is caused by the absence of incentives to work.
- In a capitalist system, production is high, but only few people work and only very few get rights to consumption. The rest die.
- A democratic system closely mimics the capitalist system. The two are factually indistinguishable. Capitalism is a necessary condition for political freedom[20] which can be inverted; political freedom will lead to capitalism.
- (Only) in a dictatorial system can a benevolent king impose minimum wages and these enable approaching the goal of Utilitarianism, with consumption for the masses optimized (without them having to work for it).

These conclusion are reached by reducing the system to the barest of minimums. The question is how realistic is this? All the more so since it only addresses the labor market, while there are also other agents in an economy and society. Some doubts can be placed on the validity of the simplistic assumptions:

- Do all workers have all the information about incomes of everybody? Is our efficient market hypothesis (EMH)[29] correct?
- Not all workers necessarily have employment and interfering in income distribution (like minimum wage) can have effects on employment[27]
- Do workers have freedom? And is this freedom coupled to the income inequality?[6]
- (In capitalism) capital also gets rewarded. There will be people owning the means of production that will not (have to) work at all and still get rich. They are playing on another board, in another vector field.
- Workers are not only incentivated by their direct neighbors, but also by people further up and down the income ladder.
- Not all people are equal and equally informed. Some simply have more qualities and produce more with the same incentive.
- Not all people are incentivated into work by income (only).
- Productivity is not a linear function of income difference.
- Income is not something that can be disconnected entirely from production, as was assumed here. In most cases, incomes are a function of productivity.

Especially this last point is worth to study further. We had set out to investigate if we can come up with an outcome of a scalable function for the distribution, as would be in line with the economic ideas of Nassim Nicholas Taleb, especially his book Antifragile[23]. Introducing feedback in the system – income depending on productivity and productivity depending on income (derivative) – is also worth to study. Yet, we find these results interesting enough and believe they may help in the eternal discussion of wealth (re)distribution.

Acknowledgment

This work is an independent work of the author. It received no funding, other than the salary at his university. The author is not member of any NGO or paid otherwise.

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