Consequent Quantum Mechanics by Applying 8-Dimensional Spinors in the Dirac equation

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ABSTRACT

Aims: A consequent quantum mechanics was developed by rendering operators also for the charge and rest mass. In this formalism the Dirac equation was extended by applying 8-dimensional spinors for the decomposition of square root in the covariant equation of special relativity.

Results: The charge and mass operators defined by 8–dimensional spinors commute with the Hamiltonian of electron and positron in electromagnetic field, but they do not commute for neutrino and quarks.

Conclusions: For neutrino the expectation values of the rest mass and charge are zero allowing these particles moving with the speed of light. The momentum of neutrino commutes with the Hamiltonian thus it has a well-defined value for the three types of neutrinos explaining why the neutrinos can oscillate. For quarks neither the rest mass nor the charge operators commute with the Hamiltonian, thus the fractional charge and renormalized mass can be considered as expectation values in the hadron states. Since any charge measurements should give eigenvalues of its operator, no fractional charge can be detected excluding possibility of observing free quarks.

1. INTRODUCTION

The intrinsic properties of elementary fermions (rest mass, charge, spin) can be different whether the weak and strong interactions are present or not. This distinction is of importance when we speak about the questions why free quarks cannot be observed or how the neutrinos can oscillate, since under these conditions only the electromagnetic interaction is present. The relativistic Dirac equation [1] gives perfect description for the electromagnetic properties of electron. This equation is derived from the operator transcription of the covariant rule of special relativity given by Klein and Gordon [2, 3]. In order to obtain a set of linear differential equation Dirac decomposed the square root of energy expression by applying four dimensional spinors. Since the covariant rule of Lorentz symmetry consists of the square of total energy, kinetic energy and rest energy, respectively, the solution of Dirac equation preserves the ambiguities of squares: any of the terms could be either negative or positive for satisfying this relation. The inherent ambiguity explains why the solution yields to both positive and negative energies. Dirac proposed a hole theory [4] to prevent electrons falling into the infinitely deep negative energy state. In his suggestion all negative energy

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states are already occupied and the Pauli exclusion principle prevents any transition to the negative energy states. His assumption was seemingly supported by the discovery of positron by Anderson [5]. The appearance of negative energy, however, can be explained straightforwardly by the symmetry of time inversion in the covariant relation, since the energy is defined by the operator of time derivative. Returning to the past is obviously forbidden, thus the electron cannot jump to the negative energy states, as it is equivalent to the inversion of time evolution.

There is a second ambiguity of covariant rule, which leads to the concept of spin as an intrinsic property of the electron in the Dirac formalism. The two spin states are converted each other also by time inversion, but this inversion affects only the intrinsic state of particles and not the overall time evolution of the state function. It means that we have to distinguish two aspects of time inversion in relativistic quantum mechanics affecting the intrinsic spin states on the one hand and the sign alternative of energy on the other hand.

There is, however, a third inherent ambiguity of the covariant rule when the energy is expressed by a square root. We connect this question of ambiguity to the definition of electric charge, which is aimed to extend the Dirac equation of electron to all elementary fermions. In this context we can raise the question how to make an entirely consequent quantum mechanics since customary operators are introduced only for energy, momentum and angular momentum, and not for charge and rest mass, which observables are represented by constants in the formalism. In the following we suggest a decomposition of square root by applying eight dimensional spinors, which offers operators for charge and rest mass and leads to an extended definition for momentum. In this formalism both the charge and rest mass will be represented by two dimensional matrices like the spin.

In the following we will point out that this procedure helps to throw light upon the question of neutrino oscillation indicating the reason why three different neutrinos can exist even though their rest mass is zero. The other riddle of particle physics is to explain why no free quarks can be detected. The above problems justify the usefulness of a study when no weak and strong interactions are considered for the neutrinos and quarks. For the neutrinos the weak interaction plays a role only in the creation and annihilation process, but when the oscillation takes place only electromagnetic interactions can be present. For the quarks the major question is why no free quarks can exist, and this question can be settled if we investigate the consequences when the strong interaction is not effective.

Recently Marsch and Narita [6] suggested a fermion unification model based on the eight dimensional spinors to generalize the Dirac equation. These authors aimed to unify the electroweak and strong interaction by assuming complex intrinsic symmetry, but did not intend for introducing operators for the mass and charge.

2. THEORETICAL METHODOLOGY

In electromagnetic field the covariant relation of special relativity can be extended by the $\boldsymbol{\Phi}$ scalar and $\vec{A}$ vector potential, where the former gives contribution to the total, the latter to the kinetic energy. In the square root, we symbolically emphasize the ambiguities by the ± signs:

$$E = \pm\sqrt{\left(\pm\left(\vec{p} - e\vec{A}\right)\right)^2 + (\pm m_0 c^2)^2} + e\Phi \qquad (1)$$

In the Dirac formalism the antisymmetric products of four dimensional spinors and ensure the decomposition of the square root into four linear equations. We transcribe the spinors by introducing direct products of the Pauli matrices

$$\hat{\alpha} = \hat{\sigma}_x \otimes \hat{\sigma} \quad \text{and} \quad \hat{\beta} = \hat{\sigma}_z \otimes \hat{I}_2 \tag{2}$$

Here $\hat{I}_2$ denotes the two dimensional unit matrix. Note the direct product is a non-commutative operation. This transcription leads to the following four dimensional Dirac operator:

$$\hat{H}_4 = c\hat{\sigma}_x \otimes \hat{\vec{\sigma}}\left(\vec{p} - e\vec{A}\right) + \hat{\sigma}_z \otimes \hat{I}_2 m_0 c^2 + \hat{I}_2 \otimes \hat{I}_2 e\Phi \tag{3}$$

An advantage of this notation is that the relativistic rule of mass increase can be straightforwardly demonstrated. Consider the Hamiltonian without electromagnetic field when the electron moves in the x direction with the speed u:

$$\hat{H}_4 = \hat{\sigma}_x cp + \hat{\sigma}_z m_0 c^2 = \begin{pmatrix} m_0 & mu/c \\ mu/c & -m_0 \end{pmatrix} c^2 = \hat{m} c^2 \tag{4}$$

From the solution of secular equation, we can derive the usual formula of mass increase as a function of speed. In Eq. 4 the $E = mc^2$ equivalence is pointed out by the introduction of the mass matrix, in which the diagonal elements represent the rest mess in the positive and negative energy states, respectively. Later we will discuss how the rest mass can be represented by an operator.

The eight dimensional spinors can be also composed from direct products of two dimensional matrices built up from the Pauli matrices. For this purpose, we introduce a pair of matrices having antisymmetric products:

$$\hat{H}_4 = \hat{\sigma}_x cp + \hat{\sigma}_z m_0 c^2 = \begin{pmatrix} m_0 & mu/c \\ mu/c & -m_0 \end{pmatrix} c^2 = \hat{m} c^2 \tag{5a}$$

$$\hat{\hat{U}}_{2,n} = -\hat{\sigma}_z sin\chi_n + \hat{\sigma}_x cos\chi_n \tag{5b}$$

This definition leads to, $\hat{\hat{U}}_{2,n} \cdot \hat{U}_{2,n} + \hat{U}_{2,n} \cdot \hat{\hat{U}}_{2,n} = 0$, which ensures that all cross terms vanish when the square of additive terms is formed. Here the n quantum number defines all intrinsic parameters of the different elementary particles by means of the chirality angles where n = 0,1,2,3. Operators of the charge, rest mass and momentum can be defined:

$$\text{Charge:} \quad \hat{q} = \hat{U}_{2,n} e \tag{6a}$$

$$\text{Rest mass:} \quad \hat{m}_0 = \hat{U}'_{2,n} m_0 \tag{6b}$$

$$\text{Momentum:} \quad \hat{\vec{p}} = \hat{\hat{U}}'_{2,n} \vec{p} \tag{6c}$$

Since the direct product is position sensitive, we use the prime when the operator has the first position, otherwise it is placed on the last position. By substituting the respective operators in the Dirac equation, we obtain the eight dimensional fermion equation:

$$\hat{H}_{8,n} = \hat{\hat{U}}_{2,n} \otimes \hat{\vec{\sigma}} \otimes c\left(\hat{I}_2 \vec{p} - \hat{U}_{2,n} e\vec{A}\right) - \hat{U}_{2,n} \otimes \hat{I}_2 \otimes \hat{I}_2 m_0 c^2 + \hat{I}_2 \otimes \hat{I}_2 \otimes \hat{U}_{2,n} e\Phi \tag{7}$$

This equation defines the charge, rest mass and momentum for the different elementary particles as an expectation value of the respective operators. We apply a convention rendering a positive eigenvalue to particles and negative to antiparticles. The key question is whether the particular physical observables are defined by diagonal or non-diagonal matrices.

3. RESULTS AND DISCUSSIONS

3.1. *Electron and positron*

The particle quantum number n = 3 corresponds to the energy equation of electron and positron given by Dirac.

$$\hat{q}_3 = \begin{pmatrix} -e & 0 \\ 0 & e \end{pmatrix}; \hat{m}_{03} = \begin{pmatrix} m_0 & 0 \\ 0 & -m_0 \end{pmatrix}; \hat{p}_3 = \begin{pmatrix} 0 & p \\ p & 0 \end{pmatrix} \quad (8)$$

Since the charge and rest mass operators are diagonal, these quantities are intrinsic parameters of the particles, or in other words, the electron and positron have charge and mass eigenstates. The momentum is represented, however, by off-diagonal matrix, it indicates that the momentum depends on the choice of inertial system.

3.2. Neutrinos

The n = 0 quantum number describes the neutrinos, when the expectation values of charge and rest mass are zero.

$$\hat{q}_0 = \begin{pmatrix} 0 & e \\ e & 0 \end{pmatrix}; \hat{m}_{00} = \begin{pmatrix} 0 & m_0 \\ m_0 & 0 \end{pmatrix}; \hat{p}_0 = \begin{pmatrix} p & 0 \\ 0 & -p \end{pmatrix} \quad (9)$$

The zero rest mass explains why neutrinos can travel with the speed of light. The momentum is, however, diagonal, that is the neutrinos are in momentum eigenstates. The three different neutrinos can have different intrinsic momentum explaining why they constitute different particles even though the rest mass is zero. This assignment can obviate the need of rest mass for explaining the neutrino oscillation [7]. The fermion equation includes the m_0 oscillating mass, which is a free parameter for the neutrinos.

3.3. Quarks

The n = 1 and 2 values represent the up and down quarks with the charge ±1/3e and ±2/3e, respectively:

$$\hat{q}_1 = \begin{pmatrix} -1/3 & \sqrt{8}/3 \\ \sqrt{8}/3 & 1/3 \end{pmatrix} e \quad (10a)$$

$$\hat{q}_2 = \begin{pmatrix} 2/3 & \sqrt{5}/3 \\ \sqrt{5}/3 & -2/3 \end{pmatrix} e \quad (10b)$$

In this case neither the charge nor the rest mass operator is diagonal that is the quarks cannot be in mass and charge eigenstates in the hadrons. The quarks have fractional charge, which is interpreted as the expectation value of in the hadron state, furthermore their renormalized mass is also represented by the respective expectation value.

Quantum mechanics gives a probability distribution by the state function before measurement, but the measurement selects only one of the possible eigenvalues. It is called as a reduction of wavefunction [8]. It means that before detection, we can give the probability distribution of.charge and mass by the hadron state function of quarks, but when any free particle is detected its charge should be either +e or –e, but never a partial charge. It explains that no free quarks can be detected in accordance to the confinement principle [9]. The quarks can exist in bonded state, where the strong interaction between quarks distributes the charge of the constituents of hadrons. It is in accordance that the gluons posses not only color, but also charge.

4. CONCLUSIONS

The covariant relation based on the Lorentz symmetry can be used for developing a consequent relativistic quantum mechanics. In this formalism the charge and rest mass are represented by operators. The operator formalism leads to a new interpretation why the neutrino oscillation can occur even though these particles have zero rest mass, it is also explained why no free quarks can be detected with fractional charges. This interpretation of neutrino oscillation and quark confinement justifies efforts for developing a general fermion equation. Though at this stage of theory no color charge was taken into account, and thus the suggested fermion equation is not considered as a complete description of hadron state, nevertheless the conclusions for the free quark are still valid, since in this state no strong interaction exists and the color charge has no relevance for describing any properties of free quarks. The fractional charge of a particle can be attributed to the strong interaction between two or three quarks, while the zero charge and rest mass for neutrinos are created by the weak interaction in the transformation processes of fermions. A multi-particle theory based on the strong and weak interaction can describe the quantum states in which these properties are given as expectation values of the charge and rest mass operators.

In the next stage of work the inclusion of strong interaction is planned including the color charge. For this purpose the Casimir invariant [10] can help to generalize further the mass, charge and spin operators.

: *Keywords: Neutrino oscillations, quark confinement, relativistic quantum mechanics*

COMPETING INTERESTS

"Authors have declared that no competing interests exist.".

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