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Consequent Quantum Mechanics by Applying 8-Dimensional Spinors in the Dirac equation

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ABSTRACT

Aims: A consequent quantum mechanics was developed by rendering operators also for the charge and rest mass. In this formalism the Dirac equation was extended by applying 8-dimensional spinors for the decomposition of square root in the covariant equation of special relativity.

Results: The charge and mass operators defined by 8-dimensional spinors commute with the Hamiltonian of electron and positron in electromagnetic field, but they do not commute for neutrino and quarks.

Conclusions: For neutrino the expectation values of the rest mass and charge are zero allowing these particles moving with the speed of light. The momentum of neutrino commutes with the Hamiltonian thus it has a well-defined value for the three types of neutrinos explaining why the neutrinos can oscillate. For quarks neither the rest mass nor the charge operators commute with the Hamiltonian, thus the fractional charge and renormalized mass can be considered as expectation values in the hadron states. Since any charge measurements should give eigenvalues of its operator, no fractional charge can be detected excluding possibility of observing free quarks.

1. INTRODUCTION

The intrinsic properties of elementary fermions (rest mass, charge, spin) can be different whether the weak and strong interactions are present or not. This distinction is of importance when we speak about the questions why free quarks cannot be observed or how the neutrinos can oscillate, since under these conditions only the electromagnetic interaction is present. The relativistic Dirac equation [1] gives perfect description for the electromagnetic properties of electron. This equation is derived from the operator transcription of the covariant rule of special relativity given by Klein and Gordon [2, 3]. In order to obtain a set of linear differential equation Dirac decomposed the square root of energy expression by applying four dimensional spinors. Since the covariant rule of Lorentz symmetry consists of the square of total energy, kinetic energy and rest energy, respectively, the solution of Dirac equation preserves the ambiguities of squares: any of the terms could be either negative or positive for satisfying this relation. The inherent ambiguity explains why the solution yields to both positive and negative energies. Dirac proposed a hole theory [4] to prevent electrons falling into the infinitely deep negative energy state. In his suggestion all negative energy

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34 states are already occupied and the Pauli exclusion principle prevents any transition to the
35 negative energy states. His assumption was seemingly supported by the discovery of
36 positron by Anderson [5]. The appearance of negative energy, however, can be explained
37 straightforwardly by the symmetry of time inversion in the covariant relation, since the energy
38 is defined by the operator of time derivative. Returning to the past is obviously forbidden,
39 thus the electron cannot jump to the negative energy states, as it is equivalent to the
40 inversion of time evolution.

41 There is a second ambiguity of covariant rule, which leads to the concept of spin as an
42 intrinsic property of the electron in the Dirac formalism. The two spin states are converted
43 each other also by time inversion, but this inversion affects only the intrinsic state of particles
44 and not the overall time evolution of the state function. It means that we have to distinguish
45 two aspects of time inversion in relativistic quantum mechanics affecting the intrinsic spin
46 states on the one hand and the sign alternative of energy on the other hand.

47 There is, however, a third inherent ambiguity of the covariant rule when the energy is
48 expressed by a square root. We connect this question of ambiguity to the definition of
49 electric charge, which is aimed to extend the Dirac equation of electron to all elementary
50 fermions. In this context we can raise the question how to make an entirely consequent
51 quantum mechanics since customary operators are introduced only for energy, momentum
52 and angular momentum, and not for charge and rest mass, which observables are
53 represented by constants in the formalism. In the following we suggest a decomposition of
54 square root by applying eight dimensional spinors, which offers operators for charge and rest
55 mass and leads to an extended definition for momentum. In this formalism both the charge
56 and rest mass will be represented by two dimensional matrices like the spin.

57 In the following we will point out that this procedure helps to throw light upon the question of
58 neutrino oscillation indicating the reason why three different neutrinos can exist even though
59 their rest mass is zero. The other riddle of particle physics is to explain why no free quarks
60 can be detected. The above problems justify the usefulness of a study when no weak and
61 strong interactions are considered for the neutrinos and quarks. For the neutrinos the weak
62 interaction plays a role only in the creation and annihilation process, but when the oscillation
63 takes place only electromagnetic interactions can be present. For the quarks the major
64 question is why no free quarks can exist, and this question can be settled if we investigate
65 the consequences when the strong interaction is not effective.

66 Recently Marsch and Narita [6] suggested a fermion unification model based on the eight
67 dimensional spinors to generalize the Dirac equation. These authors aimed to unify the
68 electroweak and strong interaction by assuming complex intrinsic symmetry, but did not
69 intend for introducing operators for the mass and charge.

70 2. THEORETICAL METHODOLOGY

71 In electromagnetic field the covariant relation of special relativity can be extended by the Φ
72 scalar and \vec{A} vector potential, where the former gives contribution to the total, the latter to
73 the kinetic energy. In the square root, we symbolically emphasize the ambiguities by the \pm
74 signs:

$$75 \quad E = \pm \sqrt{\left(\pm(\vec{p} - e\vec{A})\right)^2 + (\pm m_0 c^2)^2} + e\Phi \quad (1)$$

76 In the Dirac formalism the antisymmetric products of four dimensional spinors and ensure
 77 the decomposition of the square root into four linear equations. We transcribe the spinors by
 78 introducing direct products of the Pauli matrices

$$79 \quad \hat{\alpha} = \hat{\sigma}_x \otimes \hat{\sigma} \quad \text{and} \quad \hat{\beta} = \hat{\sigma}_z \otimes \hat{t}_2 \quad (2)$$

80 Here \hat{t}_2 denotes the two dimensional unit matrix. Note the direct product is a non-
 81 commutative operation. This transcription leads to the following four dimensional Dirac
 82 operator:

$$83 \quad \hat{H}_4 = c\hat{\sigma}_x \otimes \hat{\sigma}(\vec{p} - e\vec{A}) + \hat{\sigma}_z \otimes \hat{t}_2 m_0 c^2 + \hat{t}_2 \otimes \hat{t}_2 e\Phi \quad (3)$$

84 An advantage of this notation is that the relativistic rule of mass increase can be
 85 straightforwardly demonstrated. Consider the Hamiltonian without electromagnetic field
 86 when the electron moves in the x direction with the speed u:

$$87 \quad \hat{H}_4 = \hat{\sigma}_x cp + \hat{\sigma}_z m_0 c^2 = \begin{pmatrix} m_0 & mu/c \\ mu/c & -m_0 \end{pmatrix} c^2 = \hat{m}c^2 \quad (4)$$

88 From the solution of secular equation, we can derive the usual formula of mass increase as
 89 a function of speed. In Eq. 4 the $E = mc^2$ equivalence is pointed out by the introduction of the
 90 mass matrix, in which the diagonal elements represent the rest mass in the positive and
 91 negative energy states, respectively. Later we will discuss how the rest mass can be
 92 represented by an operator.

93 The eight dimensional spinors can be also composed from direct products of two
 94 dimensional matrices built up from the Pauli matrices. For this purpose, we introduce a pair
 95 of matrices having antisymmetric products:

$$96 \quad \hat{H}_4 = \hat{\sigma}_x cp + \hat{\sigma}_z m_0 c^2 = \begin{pmatrix} m_0 & mu/c \\ mu/c & -m_0 \end{pmatrix} c^2 = \hat{m}c^2 \quad (5a)$$

$$97 \quad \hat{U}_{2,n} = -\hat{\sigma}_z \sin\chi_n + \hat{\sigma}_x \cos\chi_n \quad (5b)$$

98 This definition leads to, $\hat{U}_{2,n} \cdot \hat{U}_{2,n} + \hat{U}_{2,n} \cdot \hat{U}_{2,n} = 0$, which ensures that all cross terms vanish
 99 when the square of additive terms is formed. Here the n quantum number defines all intrinsic
 100 parameters of the different elementary particles by means of the chirality angles where n =
 101 0,1,2,3. Operators of the charge, rest mass and momentum can be defined:

$$102 \quad \text{Charge: } \hat{q} = \hat{U}_{2,n} e \quad (6a)$$

$$103 \quad \text{Rest mass: } \hat{m}_0 = \hat{U}'_{2,n} m_0 \quad (6b)$$

$$104 \quad \text{Momentum: } \hat{p} = \hat{U}'_{2,n} \vec{p} \quad (6c)$$

105 Since the direct product is position sensitive, we use the prime when the operator has the
 106 first position, otherwise it is placed on the last position. By substituting the respective
 107 operators in the Dirac equation, we obtain the eight dimensional fermion equation:

$$108 \quad \hat{H}_{8,n} = \hat{U}_{2,n} \otimes \hat{\sigma} \otimes c(\hat{t}_2 \vec{p} - \hat{U}_{2,n} e\vec{A}) - \hat{U}_{2,n} \otimes \hat{t}_2 \otimes \hat{t}_2 m_0 c^2 + \hat{t}_2 \otimes \hat{t}_2 \otimes \hat{U}_{2,n} e\Phi \quad (7)$$

109 This equation defines the charge, rest mass and momentum for the different elementary
 110 particles as an expectation value of the respective operators. We apply a convention
 111 rendering a positive eigenvalue to particles and negative to antiparticles. The key question is
 112 whether the particular physical observables are defined by diagonal or non-diagonal
 113 matrices.

114 3. RESULTS AND DISCUSSIONS

115 3.1. *Electron and positron*

116 The particle quantum number $n = 3$ corresponds to the energy equation of electron and
 117 positron given by Dirac.

$$118 \quad \hat{q}_3 = \begin{pmatrix} -e & 0 \\ 0 & e \end{pmatrix}; \hat{m}_{03} = \begin{pmatrix} m_0 & 0 \\ 0 & -m_0 \end{pmatrix}; \hat{p}_3 = \begin{pmatrix} 0 & p \\ p & 0 \end{pmatrix} \quad (8)$$

119 Since the charge and rest mass operators are diagonal, these quantities are intrinsic
 120 parameters of the particles, or in other words, the electron and positron have charge and
 121 mass eigenstates. The momentum is represented, however, by off-diagonal matrix, it
 122 indicates that the momentum depends on the choice of inertial system.

123 3.2. Neutrinos

124 The $n = 0$ quantum number describes the neutrinos, when the expectation values of charge
 125 and rest mass are zero.

$$126 \quad \hat{q}_0 = \begin{pmatrix} 0 & e \\ e & 0 \end{pmatrix}; \hat{m}_{00} = \begin{pmatrix} 0 & m_0 \\ m_0 & 0 \end{pmatrix}; \hat{p}_0 = \begin{pmatrix} p & 0 \\ 0 & -p \end{pmatrix} \quad (9)$$

127 The zero rest mass explains why neutrinos can travel with the speed of light. The
 128 momentum is, however, diagonal, that is the neutrinos are in momentum eigenstates. The
 129 three different neutrinos can have different intrinsic momentum explaining why they
 130 constitute different particles even though the rest mass is zero. This assignment can obviate
 131 the need of rest mass for explaining the neutrino oscillation [7]. The fermion equation
 132 includes the m_0 oscillating mass, which is a free parameter for the neutrinos.

133 3.3. Quarks

134 The $n = 1$ and 2 values represent the up and down quarks with the charge $\pm 1/3e$ and $\pm 2/3e$,
 135 respectively:

$$136 \quad \hat{q}_1 = \begin{pmatrix} -1/3 & \sqrt{8}/3 \\ \sqrt{8}/3 & 1/3 \end{pmatrix} e \quad (10a)$$

$$137 \quad \hat{q}_2 = \begin{pmatrix} 2/3 & \sqrt{5}/3 \\ \sqrt{5}/3 & -2/3 \end{pmatrix} e \quad (10b)$$

138 In this case neither the charge nor the rest mass operator is diagonal that is the quarks
 139 cannot be in mass and charge eigenstates in the hadrons. The quarks have fractional
 140 charge, which is interpreted as the expectation value of in the hadron state, furthermore their
 141 renormalized mass is also represented by the respective expectation value.

142 Quantum mechanics gives a probability distribution by the state function before
143 measurement, but the measurement selects only one of the possible eigenvalues. It is called
144 as a reduction of wavefunction [8]. It means that before detection, we can give the probability
145 distribution of charge and mass by the hadron state function of quarks, but when any free
146 particle is detected its charge should be either $+e$ or $-e$, but never a partial charge. It
147 explains that no free quarks can be detected in accordance to the confinement principle [9].
148 The quarks can exist in bonded state, where the strong interaction between quarks
149 distributes the charge of the constituents of hadrons. It is in accordance that the gluons
150 possess not only color, but also charge.

151 **4. CONCLUSIONS**

152 The covariant relation based on the Lorentz symmetry can be used for developing a
153 consequent relativistic quantum mechanics. In this formalism the charge and rest mass are
154 represented by operators. The operator formalism leads to a new interpretation why the
155 neutrino oscillation can occur even though these particles have zero rest mass, it is also
156 explained why no free quarks can be detected with fractional charges. This interpretation of
157 neutrino oscillation and quark confinement justifies efforts for developing a general fermion
158 equation. Though at this stage of theory no color charge was taken into account, and thus
159 the suggested fermion equation is not considered as a complete description of hadron state,
160 nevertheless the conclusions for the free quark are still valid, since in this state no strong
161 interaction exists and the color charge has no relevance for describing any properties of free
162 quarks. The fractional charge of a particle can be attributed to the strong interaction between
163 two or three quarks, while the zero charge and rest mass for neutrinos are created by the
164 weak interaction in the transformation processes of fermions. A multi-particle theory based
165 on the strong and weak interaction can describe the quantum states in which these
166 properties are given as expectation values of the charge and rest mass operators.

167 In the suggested formalism the neutrino masses are represented by two dimensional
168 matrices where only the diagonal elements are zero in the applied basis, but they have non-
169 zero off diagonal elements. Diagonalisation of the mass matrices can offer non-zero mass
170 eigenvalues for the different neutrinos which is in full agreement with the very important
171 results of Kajita T [10] and McDonald AB [11], who discovered the neutrino oscillation. In the
172 next stage of work the inclusion of strong interaction is planned including the color charge.
173 For this purpose the Casimir invariant [12] can help to generalize further the mass, charge
174 and spin operators.

175 : *Keywords: Neutrino oscillations, quark confinement, relativistic quantum mechanics*
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177 **COMPETING INTERESTS**

178
179
180 "Authors have declared that no competing interests exist."

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