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## Dynamics of the optical pulse in a nonlinear medium: approach of moment method coupled with the fourth order Runge-Kutta method

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### Abstract

In this paper, we considered the nonlinear Schrödinger equation and applied the moment method in order to investigate the evolution of pulse parameters in nonlinear medium. This mathematical model described the effects of cubic nonlinear and the nonlinear dispersion terms on the soliton. The application of moment method leads to variational equations that is be integrated numerically by the fourth order Runge-Kutta method. The results obtained show the variations of some important parameters of the pulse namely the energy, the pulse position, the frequency shift, the chirp and the width. It reveals the effects of the nonlinear dispersion and nonlinear cubic terms on each parameter on the pulse. The moment method is appropriate to study the dynamics of the optical pulse in a nonlinear medium modeled by the nonlinear Schrödinger equation.

*Keywords: moment method, nonlinear Schrödinger equation.*

### Introduction

The generalized nonlinear Schrödinger equation (GNLSE) as a nonlinear model has been studied due to its importance in many fields of physics such a nonlinear optics, plasma physics, superconductivity, quantum mechanics [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. It's the fundamental model that governs the transmission of information through optical fibers. This equation plays the role of Newton's laws and conservation of energy in classic mechanics. In order to better understand nonlinear phenomena, it's important to solve this equation.

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In the general case, it is very difficult to find analytic solution for this equation [14]. With development of soliton theory and computer algebraic system like mathematics, much research papers has been devoted to exact solution of no linear evolution equations, especially travelling wave solution [4]. Various effective method of searching for exact solution to GNSE have been presented in the literature: the inverse scattering method, the Blacklund transformation, the Adomian method, homothopy perturbation method, the Hirota bilinear method, the Lie group method, the variable separation method, the variational iteration, the Jacobi elliptic function, the expansion method, the auxiliary equation method, the trial function method, the moment method [11, 13, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28].

The strength of these methods depends on the system that had been studied. For physical systems well defined, the choice of the method could not be uncertain. In this paper, our aim is to study the evolution of the different parameters of the pulse through optical fiber with an appropriate method. We chose the moment method because of the double possibilities it offers: one is the choice of the trial function (anstaz) according to the approximative shape of the solution and the fundamental parameters of the system, then the second is to show the influence of each nonlinearity of the system on the fundamental parameters of the soliton. In addition, since this method does not require a Lagrangian, it can be used for both dissipative and non dissipative systems.

The outline of the present paper is as follows . In section 1, we give the mathematical model. In section 2 we solve the problem by variation moment method. In section 3, we use a Gaussian function, we obtain the variational equations of the pulse parameters which are solved by the fourth order of Runge Kutta numerical method. The results and discussions are presented in section 4. Finally, we pointed out the concluding remarks.

## 1 Mathematical model

The generalized nonlinear Schrödinger equation in the dimensionless form reads [28]

$$\frac{i\partial\psi}{\partial z} + a \frac{\partial^2\psi}{\partial t^2} + ib \frac{\partial^3\psi}{\partial t^3} + c|\psi|^2\psi - i \frac{c}{\omega_0} \frac{\partial}{\partial t} (|\psi|^2\psi) = 0 \tag{1.1}$$

where  $\psi = \psi(z, t)$  is the envelop of the pulse,  $z \in [0, L]$ ,  $L > 0$  is the length of the fiber and  $t \in \mathbb{R}$  is the time,  $a$  the second order of dispersion,  $b$  the third order of dispersion,  $c$  the coefficient of self-modulation;  $\frac{c}{\omega_0}$  represents the self-steepening term. This equation holds for pulses that contain just a few optical cycles where higher nonlinear terms are included, with initial conditions  $\psi(z = 0, t)$ .

The parameters  $a, b, c$  are related to  $\beta_2, \beta_3$  as:

$$a = -\frac{\beta_2}{2}, b = -\frac{\beta_3}{6}, c = \frac{\gamma}{\omega_0}$$

## 2 Solving the problem by variational moment method

The basic idea of moment method is to treat the optical pulse like a particle whose energy  $E$ , position  $T$ , the frequency  $\Omega$ , the root mean square (RMS)  $\sigma$  and the moment related to the chirp of the pulse are defined as [3, 29, 30]:

$$E = \int_{-\infty}^{+\infty} |\psi|^2 dt ; \quad (2.1)$$

$$T = \frac{1}{E} \int_{-\infty}^{+\infty} t |\psi|^2 dt ; \quad (2.2)$$

$$\Omega = \frac{i}{2E} \int_{-\infty}^{+\infty} (\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}) dt ; \quad (2.3)$$

$$\sigma_C^2 = \frac{1}{E} \int_{-\infty}^{+\infty} (t-T)^2 |\psi|^2 dt ; \quad (2.4)$$

$$\frac{i}{2E} \int_{-\infty}^{+\infty} (\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}) dt. \quad (2.5)$$

Obviously, the evolution of these pulse parameters depend on the evolution on the pulse itself in the fiber which is governed by the NLS equation (1.1). To find the evolution of these pulse parameters, we use the equations (2.1) to (2.5) along with equation (1.1)

### Energy evolution

Differentiating (2.1) with respect to  $z$ , we have:

$$\frac{dE}{dz} = \int_{-\infty}^{+\infty} (\psi^* \frac{\partial \psi}{\partial z} + \psi \frac{\partial \psi^*}{\partial z}) dt. \quad (2.6)$$

Using (1.1) we find that:

$$\frac{\partial \psi}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 \psi}{\partial t^2} + \frac{\beta_3}{6} \frac{\partial^3 \psi}{\partial t^3} - \frac{\gamma}{\omega_0} \frac{\partial}{\partial t} (|\psi|^2 \psi) + i \gamma |\psi|^2 \psi \quad (2.7)$$

After performing calculations, we have:

$$\begin{aligned} \frac{dE}{dz} &= \int_{-\infty}^{+\infty} \left[ \frac{\beta_2}{2} \psi^* \frac{\partial^2 \psi}{\partial z^2} + \psi \frac{\partial^2 \psi^*}{\partial z^2} \right] dt + \\ &\int_{-\infty}^{+\infty} \left[ \frac{\beta_3}{6} \psi^* \frac{\partial^3 \psi}{\partial z^3} + \psi \frac{\partial^3 \psi^*}{\partial z^3} \right] dt - \\ &\frac{\gamma}{\omega_0} \int_{-\infty}^{+\infty} \left[ \psi^* \frac{\partial}{\partial t} (|\psi|^2 \psi) + \psi \frac{\partial}{\partial t} (|\psi|^2 \psi^*) \right] dt \\ &= 0 \end{aligned} \quad (2.8)$$

### Evolution of pulse position

Differentiating (2.2) with respect to  $z$  we get:

$$\frac{dT}{dz} = \frac{1}{E} \int_{-\infty}^{+\infty} t \left( \psi^* \frac{\partial \psi}{\partial z} + \psi \frac{\partial \psi^*}{\partial z} \right) dt \quad (2.9)$$

we get:

$$\begin{aligned} \frac{dT}{dz} = & i\frac{\beta_2}{2E} \int_{-\infty}^{+\infty} t \psi \frac{\partial^2 \psi^*}{\partial z^2} - \psi^* \frac{\partial^2 \psi}{\partial z^2} dt + \\ & \frac{\beta_3}{6} \int_{-\infty}^{+\infty} t \psi \frac{\partial^3 \psi^*}{\partial z^3} - \psi^* \frac{\partial^3 \psi}{\partial z^3} dt - \\ & \frac{\gamma}{\omega E} \int_{-\infty}^{+\infty} \Sigma \psi^* \frac{\partial}{\partial t} (|\psi|^2 \psi) + \psi \frac{\partial}{\partial t} (|\psi|^2 \psi^*) dt \end{aligned} \quad (2.10)$$

After integrating by parts and the definition of frequency in (2.3), we obtain:

$$\frac{dT}{dz} = \beta_2 \Omega + \frac{\beta_3}{2E} \int_{-\infty}^{+\infty} t \frac{\partial \psi}{\partial t} dt - \frac{3\gamma}{2\omega_0 E} \int_{-\infty}^{+\infty} |\psi|^4 dt \quad (2.11)$$

### Evolution of frequency schift

Differentiating (2.3) with respect to  $z$ , we have:

$$\frac{d\Omega}{dz} = \frac{i}{2E} \int_{-\infty}^{+\infty} \Sigma \frac{\partial}{\partial z} \psi^* \frac{\partial \psi}{\partial z} - \frac{\partial}{\partial z} \psi \frac{\partial \psi^*}{\partial z} dt \quad (2.12)$$

$$\frac{\partial}{\partial z} \psi^* \frac{\partial \psi}{\partial z} = \psi^* \frac{\partial^2 \psi}{\partial z \partial t} + \frac{\partial \psi^*}{\partial z} \frac{\partial \psi}{\partial t} \quad (2.13)$$

From (2.7), we can write:

$$\begin{aligned} \psi^* \frac{\partial^2 \psi}{\partial z \partial t} = & \frac{i}{2} \psi^* \frac{\partial^2 \psi}{\partial t^2} - a \psi^* \frac{\partial^4 \psi}{\partial t^4} + ib|\psi|^2 \frac{\partial}{\partial t} (|\psi|^2) + \\ & ib\psi |\psi| \frac{\partial}{\partial t} - c|\psi| \frac{\partial}{\partial t} (|\psi|^2) - c\psi |\psi| \frac{\partial}{\partial t^2} \end{aligned} \quad (2.14)$$

and

$$\begin{aligned} \frac{\partial \psi^*}{\partial z} \frac{\partial \psi}{\partial t} = & \frac{i}{2} \frac{\partial^2 \psi^*}{\partial t^2} \frac{\partial \psi}{\partial t} - a \frac{\partial^3 \psi^*}{\partial t^3} \frac{\partial \psi}{\partial t} - ib|\psi|^2 \psi \frac{\partial \psi}{\partial t} - \\ & -c \frac{\partial}{\partial t} (|\psi|^2 \psi^*) \frac{\partial \psi}{\partial t} \end{aligned} \quad (2.15)$$

Adding (2.14) and (2.15) and substituting into (2.13), we find:

$$\begin{aligned} \frac{\partial}{\partial z} \psi^* \frac{\partial \psi}{\partial z} = & \frac{i}{2} \psi^* \frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi^*}{\partial t^2} \frac{\partial \psi}{\partial t} - a \psi^* \frac{\partial^4 \psi}{\partial t^4} + \\ & \frac{\partial^3 \psi^*}{\partial t^3} \frac{\partial \psi}{\partial t} + ib|\psi|^2 \frac{\partial}{\partial t} (|\psi|^2) + \psi^* \frac{\partial \psi}{\partial t} - \psi^* \frac{\partial \psi}{\partial t} - \\ & c|\psi|^2 \frac{\partial}{\partial t} (|\psi|^2) + \psi \frac{\partial^2 \psi}{\partial t^2} + \frac{\partial \psi}{\partial t} \cdot \end{aligned} \quad (2.16)$$

Also, we can write

$$\begin{aligned}
 \frac{\partial}{\partial z} \left( \psi \frac{\partial \psi^*}{\partial z} \right) &= -\frac{i}{2} \psi \frac{\partial^2 \psi^*}{\partial t^2} - \frac{\partial^2 \psi}{\partial t^2} \frac{\partial \psi^*}{\partial t} - a \psi \frac{\partial^4 \psi^*}{\partial t^4} + \\
 \frac{\partial^3 \psi}{\partial t^3} \frac{\partial \psi^*}{\partial t} &+ ib|\psi|^2 \frac{\partial}{\partial t} (|\psi|^2) + \psi \frac{\partial \psi^*}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} - \\
 c|\psi|^2 \frac{\partial^2}{\partial t^2} (|\psi|^2) &+ \psi \frac{\partial^2 \psi^*}{\partial t^2} + \frac{\partial \psi}{\partial t} \cdot \\
 -c\psi \frac{\partial}{\partial t} (|\psi|^2) &\frac{\partial \psi^*}{\partial t}
 \end{aligned} \tag{2.17}$$

Using (2.16) and (2.17) into (2.12), we can find the evolution of frequency along the fiber to be

$$\begin{aligned}
 \frac{d\Omega}{dt} &= \frac{i}{2E} \int_{-\infty}^{+\infty} i\beta_2 \left( \frac{\partial^2 \psi^*}{\partial t^2} \frac{\partial \psi}{\partial t} + \frac{\partial \psi^2}{\partial t^2} \frac{\partial \psi^*}{\partial t} - \psi \frac{\partial^3 \psi^*}{\partial t^3} \right. \\
 &+ \psi^* \frac{\partial^3 \psi}{\partial t^3} \left. \right) dt + \frac{i}{2E} \int_{-\infty}^{+\infty} \frac{\beta_3}{6} \left( \psi^* \frac{\partial^4 \psi}{\partial t^4} - \psi \frac{\partial^4 \psi^*}{\partial t^4} + \right. \\
 &\left. - \frac{\partial^3 \psi^*}{\partial t^3} \frac{\partial \psi}{\partial t} - \frac{\partial \psi^3}{\partial t^3} \frac{\partial \psi^*}{\partial t} \right) dt - \\
 &\frac{i\bar{\gamma}}{2E\omega_0} \int_{-\infty}^{+\infty} |\psi|^2 \left( \psi^* \frac{\partial^2 \psi}{\partial t^2} - \psi \frac{\partial^2 \psi^*}{\partial t^2} \right) dt + \\
 &\frac{3i\bar{\gamma}}{2E\omega_0} \int_{-\infty}^{+\infty} \frac{\partial}{\partial t} |\psi|^2 \left( \psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) dt - \\
 &\frac{\bar{\gamma}}{E} \int_{-\infty}^{+\infty} |\psi|^2 \frac{\partial}{\partial t} |\psi|^2 dt
 \end{aligned} \tag{2.18}$$

In order to calculate  $\frac{d\Omega}{dz}$  we evaluate one by one the integrals on right hand side of the (2.18). After computation, we get:

$$\begin{aligned}
 \frac{d\Omega}{dt} &= -\frac{i\bar{\gamma}}{2E\omega_0} \int_{-\infty}^{+\infty} |\psi|^2 \left( \psi^* \frac{\partial^2 \psi}{\partial t^2} - \psi \frac{\partial^2 \psi^*}{\partial t^2} \right) dt - \\
 &\frac{3i\bar{\gamma}}{2E\omega_0} \int_{-\infty}^{+\infty} \frac{\partial}{\partial t} |\psi|^2 \left( \psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) dt - \\
 &\frac{\bar{\gamma}}{E} \int_{-\infty}^{+\infty} |\psi|^2 \frac{\partial}{\partial t} |\psi|^2 dt
 \end{aligned} \tag{2.19}$$

Rearranging (2.19) and after computation, we obtain

$$\frac{d\Omega}{dt} = -\frac{i\bar{\gamma}}{E\omega_0} \int_{-\infty}^{+\infty} \frac{\partial}{\partial t} |\psi|^2 \left( \psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) dt \tag{2.20}$$

## Evolution of chirp parameter

Let's differentiate (2.5) with respect to  $z$ , we have:

$$\frac{d\tilde{C}}{dz} = \frac{i}{2E} \int_{-\infty}^{+\infty} (t-T) \frac{\partial}{\partial z} \left( \psi^* \frac{\partial \psi}{\partial z} \Sigma - \frac{\partial}{\partial z} \left( \psi \frac{\partial \psi^*}{\partial z} \Sigma \right) \right) dt \quad (2.21)$$

From (2.16) and (2.17), we have:

$$\begin{aligned} \frac{d\tilde{C}}{dz} = & \frac{\beta_2}{4E} \int_{-\infty}^{+\infty} (t-T) \Sigma \left( -\frac{\partial^2 \psi^*}{\partial t^2} \frac{\partial \psi}{\partial t} + \frac{\partial \psi^2}{\partial t^2} \frac{\partial \psi^*}{\partial t} \right) \Sigma - \\ & - \frac{\partial^3 \psi^*}{\psi \partial t^3} + \psi^* \frac{\partial^3 \psi}{\partial t^3} + \Sigma \int dt \\ & \frac{i}{2E} \int_{-\infty}^{+\infty} (t-T) \frac{\beta_3}{6} \Sigma \left( \psi^* \frac{\partial^4 \psi}{\partial t^4} - \psi \frac{\partial^4 \psi^*}{\partial t^4} \right) \Sigma + \\ & - \frac{\partial^3 \psi^*}{\partial t^3} \frac{\partial \psi}{\partial t} - \frac{\partial \psi^3}{\partial t^3} \frac{\partial \psi^*}{\partial t} \Sigma dt - \\ & \frac{i\bar{\gamma}}{2E\omega_0} \int_{-\infty}^{+\infty} (t-T) |\psi|^2 \left( \psi^* \frac{\partial^2 \psi}{\partial t^2} - \psi \frac{\partial^2 \psi^*}{\partial t^2} \right) \Sigma dt - \\ & \frac{3i\bar{\gamma}}{2E\omega_0} \int_{-\infty}^{+\infty} (t-T) \frac{\partial}{\partial t} |\psi|^2 \left( \psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) \Sigma dt - \\ & \frac{\bar{\gamma}}{E} \int_{-\infty}^{+\infty} (t-T) |\psi|^2 \frac{\partial}{\partial t} |\psi|^2 dt \end{aligned} \quad (2.22)$$

After many integrations by parts, we get:

$$\begin{aligned} \frac{d\tilde{C}}{dz} = & \frac{\beta_2}{E} \int_{-\infty}^{+\infty} \frac{\partial \psi^2}{\partial t} dt + \frac{i\beta_3}{4E} \int_{-\infty}^{+\infty} \Sigma \left( -\frac{\partial^2 \psi^*}{\partial t^2} \frac{\partial \psi}{\partial t} - \right. \\ & \left. \frac{\partial \psi^2}{\partial t^2} \frac{\partial \psi^*}{\partial t} \right) \Sigma dt - \frac{i\bar{\gamma}}{2E\omega_0} \int_{-\infty}^{+\infty} |\psi|^2 \left( \psi \frac{\partial \psi^*}{\partial t} - \psi^* \frac{\partial \psi}{\partial t} \right) \Sigma dt \\ & \frac{3i\bar{\gamma}}{2E\omega_0} \int_{-\infty}^{+\infty} (t-T) \frac{\partial}{\partial t} |\psi|^2 \left( \psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) \Sigma dt + \\ & \frac{\bar{\gamma}}{2E} \int_{-\infty}^{+\infty} \psi^2 dt \end{aligned} \quad (2.23)$$

## Evolution of the RMS width

We differentiate (2.4) with respect to  $z$  to obtain

$$2\sigma E \frac{d\sigma}{dz} = \int_{-\infty}^{+\infty} (t-T)^2 \left( \psi^* \frac{\partial \psi}{\partial z} + \psi \frac{\partial \psi^*}{\partial z} \right) dt \quad (2.24)$$

$$\begin{aligned} 2\sigma E \frac{d\sigma}{dz} = & \frac{i\beta_2}{2} \int_{-\infty}^{+\infty} (t-T)^2 \left( \psi \frac{\partial^2 \psi^*}{\partial t^2} - \psi^* \frac{\partial^2 \psi}{\partial t^2} \right) \Sigma dt - \\ & \frac{\bar{\gamma}}{\omega_0} \int_{-\infty}^{+\infty} (t-T)^2 \left( \psi^* \frac{\partial}{\partial t} |\psi|^2 \Sigma + \psi \frac{\partial}{\partial t} (|\psi|^2 \psi^* \Sigma) \right) dt + \\ & \frac{\beta_3}{6} \int_{-\infty}^{+\infty} (t-T)^2 \left( \psi^* \frac{\partial^3 \psi}{\partial t^3} + \psi \frac{\partial^3 \psi^*}{\partial t^3} \right) \Sigma dt \end{aligned} \quad (2.25)$$

$$\frac{d\sigma}{dz} = \frac{\beta_2 c}{\sigma} + \frac{\beta_3}{2\Gamma E} \int_{-\infty}^{+\infty} (t-T) \frac{\partial \psi^2}{\partial t} dt \quad (2.26)$$

### 3 Numerical simulation with Runge-Kutta 4

Let's choose the pulse shape on the Gaussian form [31]

$$\psi(z, t) = A \exp \left[ i\phi - i\Omega(t-T) - (1+iC) \frac{(t-T)^2}{2\tau^2} \right] \quad (3.1)$$

with  $\tau^2 = K\sigma^2$ ,  $C = 2\tilde{C}$ ,  $K = cte$

We obtain a variational equations for each parameter as follows:

$$\begin{aligned} \frac{dE}{dz} &= 0 ; \\ \frac{dT}{dz} &= \beta_2 \Omega + \beta_3 / 2 \left[ \Omega^2 + \frac{1+C^2}{2\tau^2} \right] + \sqrt{\frac{3\bar{\gamma}E}{8\pi\omega_0}} ; \\ \frac{d\Omega}{dz} &= \sqrt{\frac{\bar{\gamma}EC}{2\pi\omega_0\tau^3}} ; \\ \frac{dC}{dz} &= 2\beta_2 \Omega^2 + \beta_2 \frac{1+C^2}{\tau^2} + \beta_3 \frac{1+C^2}{2\tau^2} + \sqrt{\frac{4\bar{\gamma}E\Omega}{2\pi\omega_0\tau}} + \sqrt{\frac{\bar{\gamma}E}{2\pi\tau}} ; \\ \frac{d\tau}{dz} &= \frac{\beta_2 C}{\tau} + \frac{\beta_3 \Omega C}{\tau} . \end{aligned}$$

We solved numerically the variational equations using the fourth order of Runge Kutta algorithm in Matlab [32]. The results are depicted in Figure 1. for the following values:  $\beta_2 = 0.5$ ,  $\beta_3 = 0.6$ ,  $\gamma = 2$ ,  $\omega_0 = 0.1$ . The initial conditions are given by  $E(0) = 1$ ,  $T(0) = 50$ ,  $\omega(0) = 2$ ,  $C(0) = 0$ ,  $\tau(0) = 1$ .

### 4 Discussion

$\frac{dE}{dz} = 0$ , therefore the pulse energy remains constant when the pulse propagates along the fiber.

Since the width increases (fig.5) then the pulse flattens and according to equation (2.26), this flattening is due to dispersion effects. The increasing of the chirp confirm this. The equation (2.11) shows that the pulse position is affected by any frequency shift due to the group velocity dispersion  $\beta_2$  and the third order dispersion  $\beta_3$ . As the pulse propagates, the frequency increases quickly at the beginning then reach a limit value after a distance (fig.2).

The equation (2.21) shows that the chirp is not affected by the group velocity dispersion  $\beta_2$  nor the self-steepening  $\bar{\gamma}$ . The equation (2.26) shows that the evolution of the width depends on the group velocity dispersion  $\beta_2$  and the third order dispersion  $\beta_3$ ; it's not affected by the self-steepening parameter.

The above equations for the evolution of the pulse parameter reduce the complexity of the problem but they are still not a useful form because they depend on the shape  $\psi(z, t)$ , which is not known until 1.1 is solved. If one has some knowledge of the pulse shape and its dependence on the five moments, the problem can be solved approximately. The zoom of parts of the chirp and width curves (respectively Figure 4. and Figure 6.) shows that these fundamental parameters can be modelled by an approximative linear function of z starting from a given distance. The analytic solving would be less difficult.

Let us notice that similar works was carried out by using other variational methods in particular the Lagrangian Variational and Collective Variable methods [21, 33, 34, 35]. These methods use different formalisms but lead to a set of variational equations. The integration of these differential equations finally make it possible

to appreciate the evolution of the parameters defined in the trial function. The major complexity of these methods lies in the choice of the initial conditions as well as the values of coefficients of the nonlinearities in the nonlinear Schrödinger equation in order to obtain physically solutions. They strongly vary from a method to another. They have nevertheless the common advantage to determine from the variational equations the nonlinear effects which affect each parameter in the ansatz; the appreciation of the intensity of these influences depending on the values coefficients of the nonlinear terms and thus of the intrinsic characteristics of the medium.

## Conclusion

In this paper, we used the moment method to describe the dynamics of the pulse in nonlinear medium such as optical fiber. Considering the generalized nonlinear equation, we used the moment method formalism to derive a set of ordinary differential equations which we solved with the fourth order of Runge-Kutta algorithm. Indeed, the moment method allowed us to choose the ansatz in accordance with the fundamental parameters of optical. The results obtained reveal the variations of each parameter of systems and show that the pulse flattens. The further work will be interested to the comparison of the variational methods namely the moment method, the collective method and the Langrangian variational approach using the high order nonlinear Schrödinger equation.



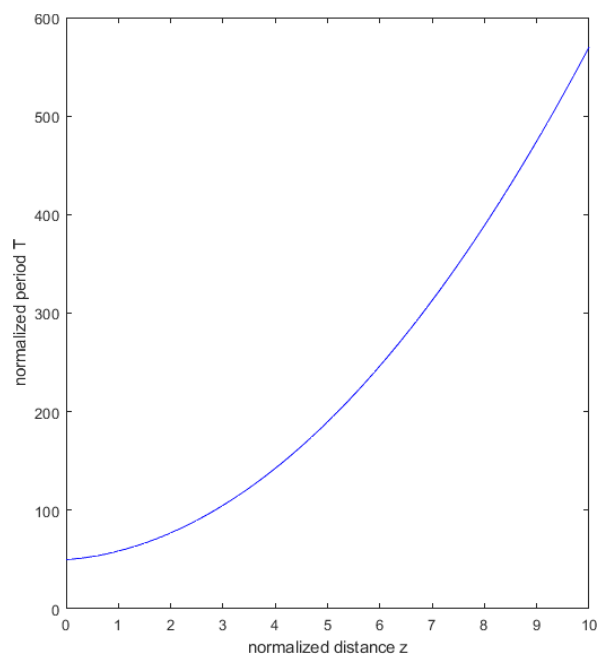


Figure 1: Variations of the period  $T$  with respect to the distance  $z$

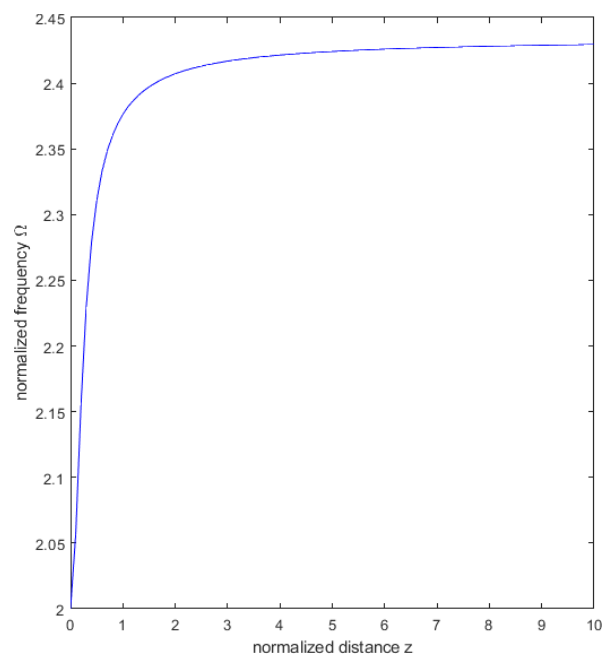


Figure 2: Variation of the frequency  $\Omega$  with respect to the distance z

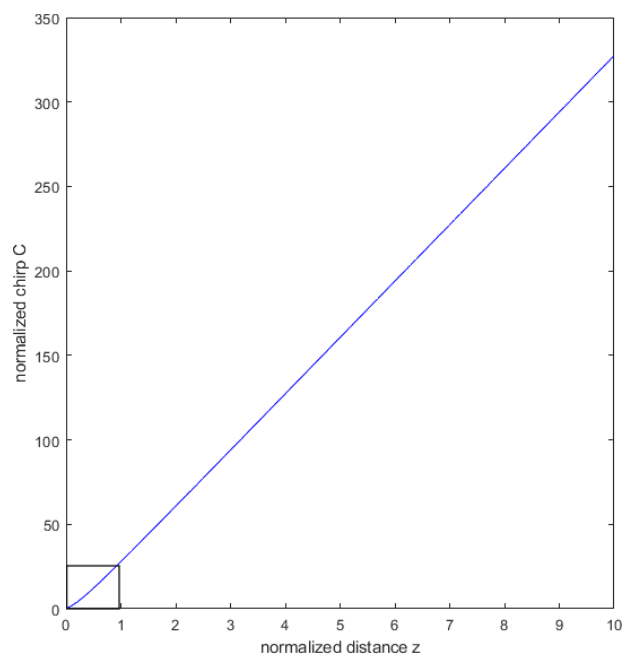


Figure 3: Variation of the chirp C with respect to the distance z

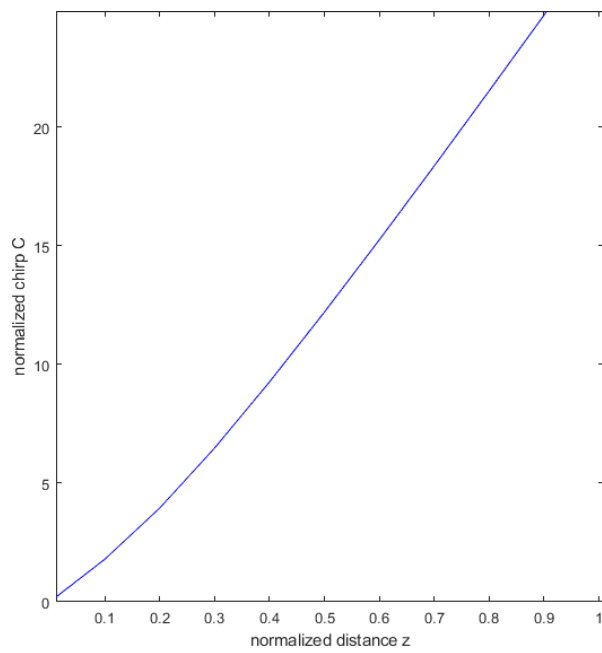


Figure 4: Zoom of the framed part of the chirp C

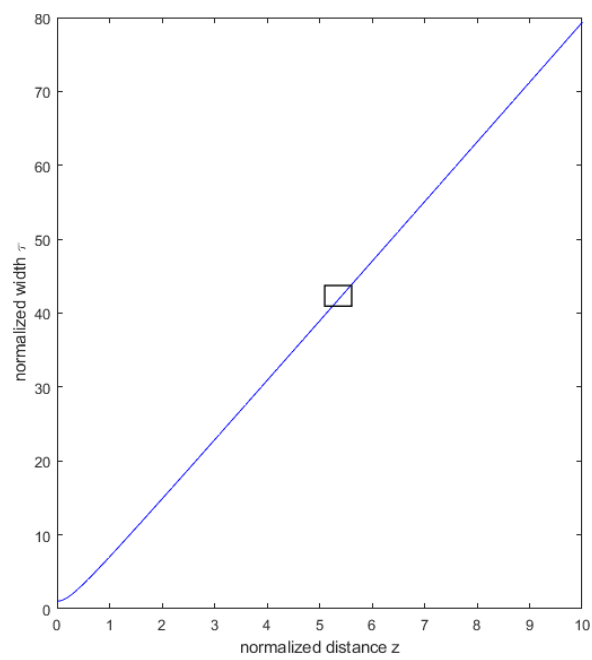


Figure 5: Variation of the width  $\tau$  with respect to the distance  $z$

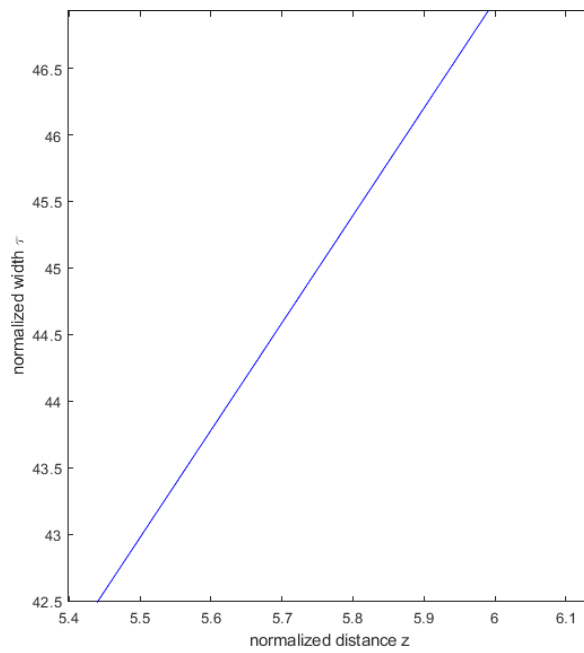


Figure 6: Zoom of the framed part of the width  $\tau$

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