LUMPED ELEMENTS AND ITS EXISTENCE IN QUASI LUMPED ELEMENT RESONATOR ANTENNA

Abstract

In this paper, the Quasi Lumped Element Resonator Antenna is reviewed. It is composed of Lumped elements. Lumped Elements are passive components whose size across any dimensions should be small to make it a lumped element. The various researches that have been done to come about the various types of basic building blocks of the lumped element are staged in this write up. This review is towards accomplishing the derivation of the component elements used in the design of the Quasi Lumped Element Resonator Antenna. These elements are the interdigital capacitor, inductor and pad capacitors. The pertinent formulae for determining each one of them were all expressed in this review. The formula for calculating the resonance frequency of the Quasi Lumped Element Resonator Antenna was expressed in this review. The equivalent circuit model for the lumped elements were all reviewed and presented. This review brings about how the lumped elements are involved in the design of the Quasi Lumped Element Resonator Antenna.

Keywords- Lumped Element, Frequency, Quasi Lumped Element Resonator Antenna.

1.0 Introduction

Devices capable of radiating and receiving radio waves are known as antennas (Balanis, 2016). They are at the interface between free space and the guiding device. The guiding device is the transmission line through which an electrical signal travels before conversion into electromagnetic waves or radiation energy for transmission or transportation through free space.

Antennas are of a variety of types with each having different shapes. These include wire antenna, microstrip patch antenna, reflector antenna, array antenna, lens antenna, aperture antenna and Quasi Lumped Element Resonator Antenna. Quasi Lumped Element Resonator Antenna is an exception in its design behavior because its resonance frequency depends on some lumped element components. These makes it to have a high degree of freedom and flexibility.

Quasi lumped Element Resonator antenna is made up of lumped capacitor, lumped inductors. By understanding the behavior of these lumped elements, the antenna behavior can be predicted. Lumped elements are passive components in microwave circuits whose size across any dimension is much smaller than the operating wavelength to ensure that there is no appreciable phase shift between the input and output terminals. When using lumped elements at RF and microwave frequencies, the maximum dimensions for these components should be about $\lambda/20$, where λ is the guide wavelength (Bahl, 2003). Lumped elements circuits can be divided as shown in Figure 1.0

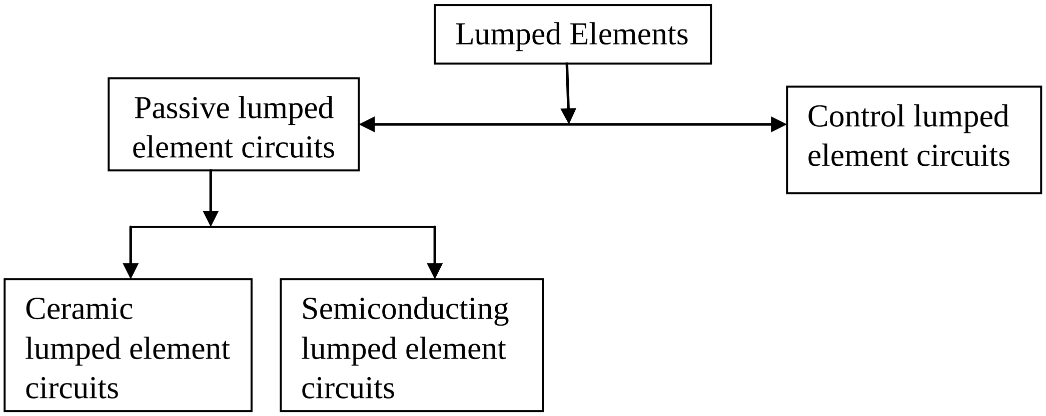


Figure 1.0: Division of Lumped Elements in Microwave Circuit

The lumped element circuit is a ceramic lumped element circuit if thick film printed inductors and discrete capacitors are used, while it is a semiconductor lumped element circuits when High Temperature Semiconductors (HTSs) are used in its design instead of the Duroid Microwave substrate (Bahl, 2003).

Figure 1.1 shows the basic building blocks from which lumped elements are derived and some commonly used components

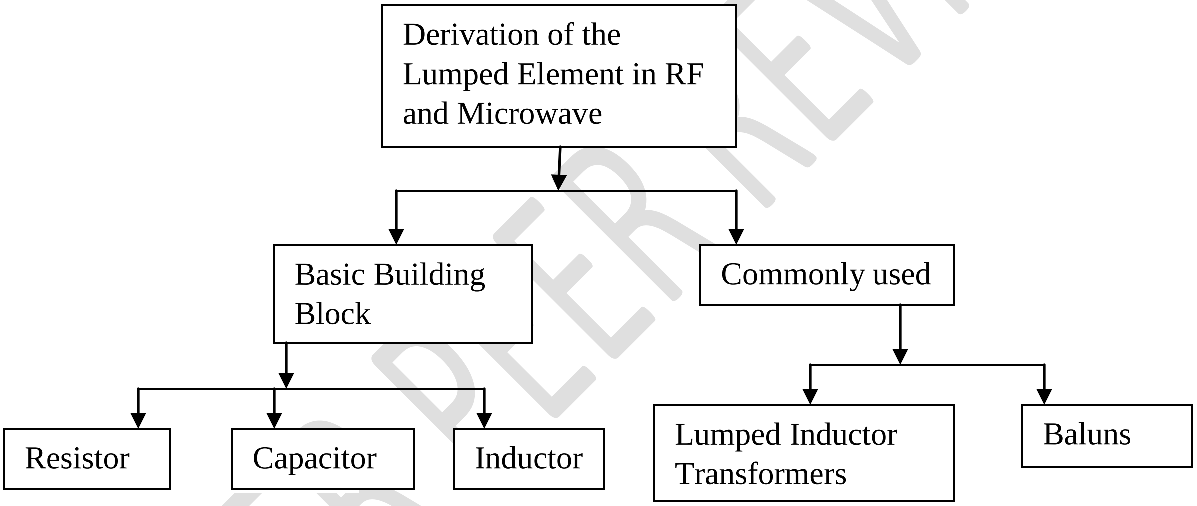


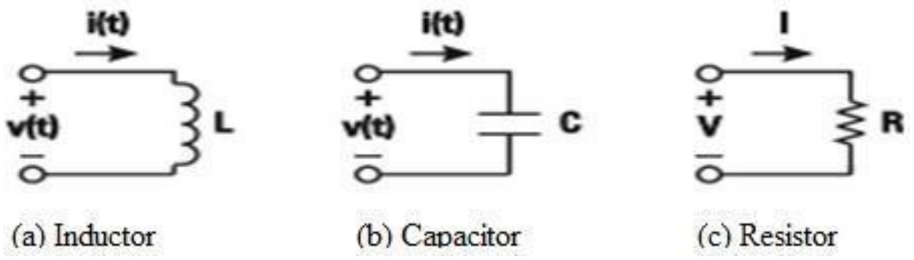
Figure 1.1: Lumped Elements in RF and Microwave Circuits

From Figure 1.1, the basic building blocks of lumped element in RF and microwave circuits are the resistor, capacitor and inductor. While the commonly used are Lumped inductor transformers and baluns (Bahl, 2003)

1.1 Basic Circuit Elements of the Lumped Element

In this subsection a brief review of the basic mathematical relationship between the terminal voltage and current across each of the circuit elements is described.

This review is necessary as it serves as the background for this study. Consider that these elements (Inductor,L; Capacitor C; and Resistor R) as shown in Figure 2.8 are ideal (pure and linear).



(a) Inductor (b) Capacitor (c) Resistor

Figure 1.3: Two-Terminal Voltage and Current Representation of Lumped Elements (Bahl, 2003).

In Figure 1.3 (a), an ideal inductor of inductance L is depicted, i(t) is the time-varying current passed through the inductor of inductance L and v(t) is voltage across its terminals. The inductor stores or releases magnetic energy W_m and does not store electric energy. The inductor does not dissipate any power and the phase of the time-varying current *i(t)* lags the phase of the voltage *v(t)* across its terminals. This is stated mathematically as follow (Bahl, 2003):

$$V_{(t)} = L\frac{di_{(t)}}{dt} \qquad (1.0)$$

$$V=jwLi \qquad (1.1)$$

$$I_{(t)} = \left(\frac{1}{L}\right)\int V_{(t)}dt \qquad (1.2)$$

$$I=\frac{v}{jwL} \qquad (1.3)$$

$$w_m=\frac{1}{2} * Li_0^2 \qquad (1.4)$$

In equations (1.0) to (1.4), the time dependence is assumed to be $e^{j\omega t}$ and io is the root mean square (rms) value of the current.

Figure 1.3(b) depicts an ideal capacitor of capacitance C. i(t) is the time-varying current passed through the capacitor C and v(t) is voltage across its terminals. The capacitor of capacitance C in Figure 1.3(b) stored or released energy but only of electric type. The capacitor does not dissipate any power and the phase of the time-varying current *i(t)* leads the phase of the voltage *v(t)* across its terminals. This is stated mathematically as follows (Bahl, 2003):

$$I_{(t)} = C * \frac{dv_{(t)}}{dt} \qquad (1.5)$$

$$I=jwcv \qquad (1.6)$$

$$V_{(t)} = \frac{1}{c} * \int i_{(t)}dt \qquad (1.7)$$

$$W_e= CV_0^2 \qquad (1.8)$$

where

v_0 is the rms value of the voltage.

A linear resistor is a lossy component whose dimensions are much less than the operating wavelength. In this component, the voltage applied to its terminals and the current passing through the resistor are in phase and the incident power is completely dissipated. Let V and I be the rms voltage and current across a resistor of value R, therefore, Ohm’s law is stated as:

$$V=RI \tag{1.9}$$

And the power dissipated is given as:

$$P=IV \tag{1.10}$$

2.0 Lumped Element Modeling

In order to use lumped elements correctly, the idea about how the lumped elements are modeled should be known. At RF and microwave frequencies, to model this lumped elements, they are realized based on a maximum dimension of one-twentieth of a wavelength, which is based on a small section of a microstrip line. The design is done by selecting a suitable length of each section. The lumped capacitor can be realized by using an open circuited ($Z_L = \infty$) microstrip sections and a lumped inductor is realizable using a short circuited ($Z_L = 0$) microstrip section. Thus, a small section of the short circuited transmission line behaves as a lumped inductor in series with a resistance R (Rafiee, 2015). It is a lumped inductor if the value of the resistance is very small, but where the conductor used in designing the lumped element is Nicr (gold), the value of the resistance is high and the lumped element is resistive. The proportional value of the resistance, inductance, and capacitance components depend on the use of the lumped element. If the short circuited line has zero resistance in series with the inductor. The short circuited microstrip line then behaves as an inductor, otherwise when the resistance is very high, as in the case of an inductor in series with a high resistance, for example when Nicr (gold) is used as the conductor, the microstrip section behaves as a resistor (Olokede, 2013).These

small sections are called quasi-lumped elements at High Frequency (HF) since the sections are used to replace the actual lumped elements (Rafiee, 2015). An ideal lumped element is not realizable at lower microwave frequencies because of the associated parasitic reactance caused by fringing fields. Each component has associated electric and magnetic fields and finite dissipative loss at RF and microwave frequencies (Rafiee, 2015).

Thus, each component stores or releases magnetic and electric energies across them and their resistance dissipates power (Bahl, 2003).

Basic circuit elements (resistance, inductance, and capacitance) with their associated parasitics are included in Lumped-Element Equivalent Circuit (LEEC) models. The relative measure of the C, L, and R components in the LEEC depend on the necessary use of the LE. The LEEC model are used to describe the electrical behavior of the components. For a Computer Aided Design (CAD) of MICs and MMICs (Leferink, 1995), there is need for a model that is comprehensive. This constitutes the effect of fringing fields, ground plane, conductor thickness, substrate material and thickness, associated mounting techniques and applications needed, non-uniform current distribution. Note that the non-uniform current distribution is caused either by the field produced by the current itself (skin-effect), the return current (proximity effect) or the current induced in the primary conductor (caused by the field of the return current on the LEs) (Rafiee, 2015).

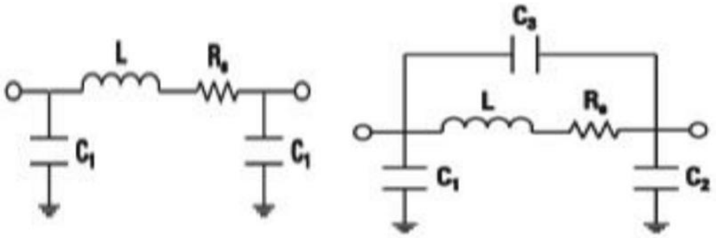
For an accurate modelling of the lumped element, an equivalent circuit representation of the lumped element LEEC together with its parasitics and the corresponding frequency depending characteristics are necessary. Thus, a LEEC model includes the required circuit elements to completely demonstrate its behaviour, particularly its possible resonances. The models can be completed using mathematical, CAD simulation and measurement based methods. The component size should be made much smaller as a lumped element is realizable at a dimension of the wavelength/20 (Bahl, 2003).

2.1 Equivalent Circuit Modelling of Lumped Inductor

Modelling in terms of electrical circuits and numerical equations for each of the components (interdigital capacitor, pad capacitor, straight line inductor or inductor) of lumped element is indispensable for a precise quasi lumped element modeling. Thus, as early as 1943, models were developed for LEs. These models were based on analytical semi empirical equations. Terman, (1943) was able to publish an expression for the inductance of a thin metallic straight line. The expression was subsequently improved by Caulton *et al*., (1968), by adding the metallization thickness. Wheeler, (1928) published an approximate formula for the inductance of a circular spiral inductor with reasonably good accuracy at lower microwave frequencies. This formula has been tremendously used in the design of microwave lumped circuits as stated in equation (2.1). Grover, (2004) discussed inductance calculations for several geometries. In a lot of the cases, two methods have been used in the theoretical modeling of microstrip inductors for MICs. The first method is the lumped-element approach and the second method is the coupled-line approach. The lumped–element approach uses frequency independent formulae for free-space inductance with ground plane effects. These frequency-independent formulae are useful only when the total length of the inductor is a little fraction of the operating wavelength and when inter-turn capacitance can be ignored. In the coupled-line approach, the multi-conductor coupled microstrip lines are used to analyze an inductor. The later technique predicts performance reasonably well for up to two turns and frequencies up to 18 GHz (Bahl, 2003).

3-D electromagnetic simulators can be used to determine the accurate characterization of inductors including the effects of radiation, surface waves, and not excluding the interaction between components on the performance of densely packed inductors in MMICs (Bahl, 2003).

One method that gives an accurate result for a model of lumped inductor is by measuring dc resistance and S-parameters. Although, they are limited to the device being measured, the equivalent circuit parameters are extracted from computer optimization and are correlated with the measured dc resistance and S-parameters data (one or two port data) and are valid up to 26 or 40 GHz depending on the application.



(a) Microstrip Section and Loop (b) Coil

Figure 2.0: Equivalent Circuit Model for Lumped Inductor (Bahl, 2003)

2.2.3.2 Equivalent Circuit Modelling of Lumped Capacitor (Interdigital Capacitor)

An interdigital capacitor is a multi finger periodic structure. This multi finger structure consists of fingers that are equally spaced with gaps of equal width in between them. The gaps are very narrow and the capacitance occurs across the interdigital fingers. The gaps are folded to form a long length and use a small amount of space and consequently its attendant relevance has a lumped element. The interdigital capacitor is larger than an overlay capacitor (Bahl, 2003). The structure for an interdigital capacitor is shown in Figure 2.1. The capacitance of the interdigital capacitor can be increased by increasing the number of fingers, putting a thin layer high dielectric constant material between the conductors and the substrate, using an overlay dielectric. This methods increase the interdigital capacitance and the overlay dielectric also acts as a protective shield after fabrication. Interdigital capacitors can be employed with Monolithic Microwave Integrated Circuit (MMIC) designs and Microwave Integrated Circuit (MIC) designs at higher frequency as they can be used instead of the discrete circuit designs. A necessary design consideration for the interdigital capacitor is to keep its size very small in comparison with a wavelength in order for it to possess the characteristics of a lumped element.

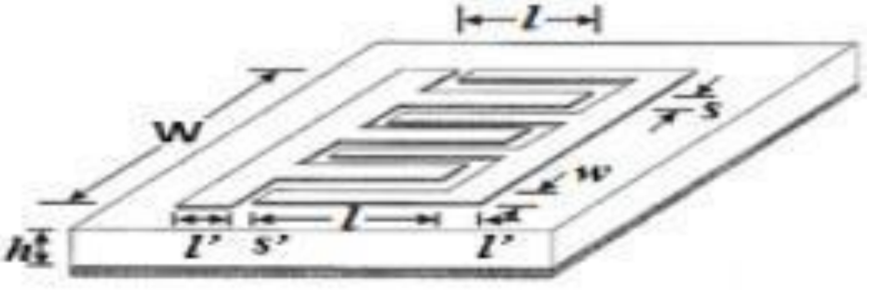


Figure2.1: Interdigital Capacitor (Bahl, 2003)

To analyze interdigital capacitors, there are four famous techniques. The techniques are approximate analysis, J-inverter network equivalent representation, full wave analysis, and measurement based model. Due to the relevance, comprehensibility, and accuracy (Bahl, 2003) of the first method, it is utilized in this work.

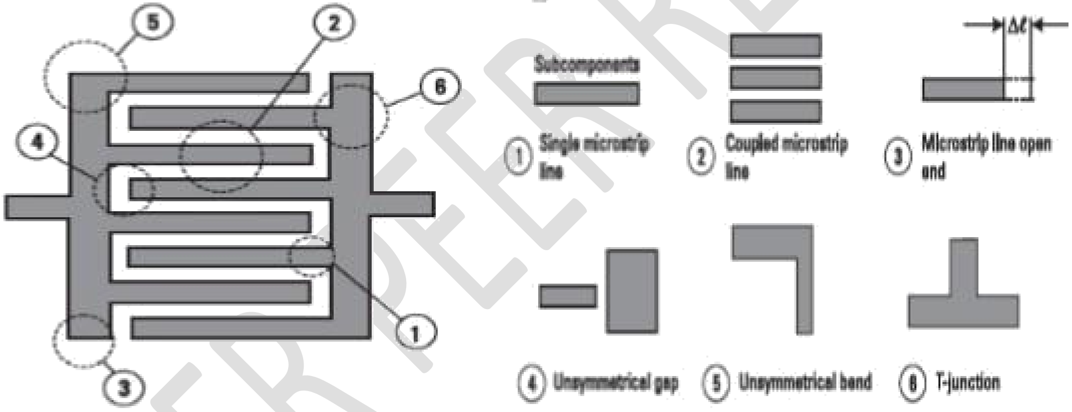
The analysis for the interdigital capacitor using the approximate analysis was based on lossless microstrip coupled line theory (Alley, 1970) and the loss effects were added to make the lossy coupled microstrip lines by representing it with mathematical calculations (Hobdell, 1979).

About four years since Hobdel, (1979) introduced the mathematical calculations; the effect of metallization was added onto the two factors: capacitance and the Q factor (Esfandiari *et al.*, 1983). On the other hand, Alley's theory did not consider different positions of fingers with admittance calculation of parallel shunt fingers.

Also, the phase shift along the main line was not considered which was distinct in interdigital capacitor with high number of fingers. In addition, the effects of gaps, bends, the T-junctions, the open end of microstrip line and discontinuity of the structure were not taken into account. A more accurate characterization of these capacitors could be performed if the capacitor geometry was divided into basic microstrip sections and subsections (Pettenpaul *et al.*, 1988).ThePattenpaul *et al*. (1988) publication was based on (Wolff and Kibuuka, 1984) theory. This model could therefore be said to provide better accuracy than the previously reported analyses. Nonetheless, this method can at best be regarded as an approximate solution rather than an explicit or exact. This is due to several assumptions in the grouping of sub-sections and as such, could not account for interaction effects between the microstrip sections. This microstrip sections and subsections are:

1. The single microstrip line.
2. The coupled microstrip lines.
3. The microstrip open-end discontinuity.
4. The microstrip unsymmetrical gap.
5. The unsymmetrical microstrip 90^0 bend.
6. The microstrip T-junction discontinuities.

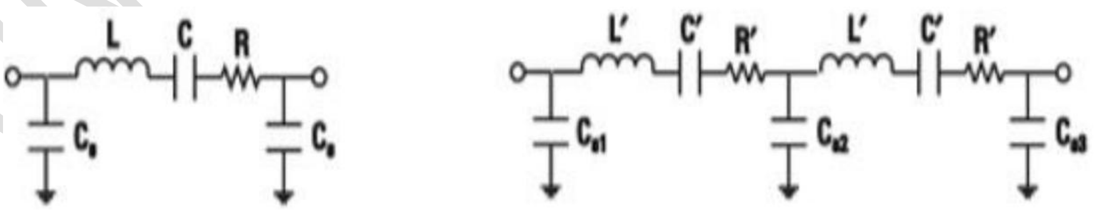
These components are shown in Figure 2.2.



(a) Interdigital Capacitor (b) Subcomponents

Figure 2.2: Lumped Capacitor (Bahl, 2003)

The value of the interdigital capacitor can be obtained using equation (2.2)



(a) Low Frequency (b) High Frequency

Figure 2.3: Lumped Element Equivalent Circuit Models of Interdigital Capacitor (Bahl, 2003).

Numerical Approach: Although, the equivalent circuit models for the interdigital capacitor and inductor can be used to model them, they were explained as not exact and explicit. Equations (2.1) through (2.6) can be used to determine the dimensions and resonance frequency required for a particular design of the quasi lumped element resonator antenna. However, these are just estimates, especially at high frequencies, for designing the circuit and are not accurate. By using numerical methods employed via electromagnetic (EM) simulators, the antenna can be simulated efficiently and additional features in the layout of the design can be adjusted, it can meet with different configurations (2-D or 3-D) and it is versatile.

The simulators put into consideration other constraints such as junction discontinuities, substrate effects (thickness and dielectric constant). The most common field solver technique employed with planar structures is the Method of Moments (MoM) while for 3-D structures, it is more of the Finite Element Method (FEM). Both of these techniques function with EM analysis in the frequency domain. FEM is used when a more complex design is involved than with MoM and it also uses a lot of computation time and requires much more memory. EM analysis in the time domain also exists, example of these techniques are Transmission Line Matrix (TLM) method and the Finite Difference Time-Domain (FDTD) method.

From the litteratures, it is now known that the inductor and capacitors are given by

i. **Inductor:** The inductor is a single, narrow and straight conductor, which is located at the centre finger and shorted across the interdigital capacitor. It is presented in parallel with the interdigital capacitor in the circuit diagram for the quasi lumped element resonator antenna showed in Figure2.1. The magnitude of the inductor can be increased by using a meander line. The equation fordetermining the value of the inductance of the inductor is given as (Avenhaus, 1996) :

$$L = 200 * 10^{-9} I_L \left(In \frac{2*I_L}{w_1+t} + 0.50049 + \frac{w_1}{3*I_L} \right) \qquad (2.1)$$

where:

L is the inductance of the interdigital capacitor,

I_L is the inductor length

W_1 is the inductor width

t is the thickness of the resonator

ii. **Interdigital capacitor:** It is a structure containing a multi periodic arrangement of conductors with equal spacing in between in the form of a multi finger like periodic pattern. This structure is associated with the capacitance used by the quasi lumped element resonator antenna. The interdigital capacitor is usually in parallel with the inductor in the quasi lumped element resonator antenna equivalent circuit (Ain *et al.*, 2013). The approximate value for the interdigital capacitor can be calculated (Huang *et al.*, 1999) as follows:

$$C = e_o * \frac{e_r+1}{2} * (N - \Delta) * C_L \qquad (2.2)$$

where:

C is the capacitance of the interdigital capacitor

e_o is the permittivity of free spaced

e_r is the dielectric constant of the substrate

N is the number of fingers

Δ is the correction factor

C_L is the overlapping length of the interdigital capacitor fingers.

iii. **Pad Capacitors:** The pad capacitors are two and are at both sides of the resonator. They are represented with Cp1 and Cp2 as shown on Figure 2.1(b). They act as capacitors to the ground and can be adjusted to tune the resonant frequency of the resonator. The respective values of Cp1 and Cp2 do not depend on the number of interdigital fingers or the size of the fingers. This is because the location of the ground plane can be adjusted to minimize their effects. The two pad capacitors are of the same value and their value can be estimated (Bogatin, 1988) as:

$$C_P = \left[\frac{2.85 * e_{eff}}{In(1+\frac{1}{2}*\left(\frac{8*\square}{w_{eff}}\right)*((\frac{8*\square}{w_{eff}})+\sqrt{(\frac{8*\square}{w_{eff}})^2+\pi^2})}\right] * \left[\frac{l}{25.4*10^{-3}}\right] \quad (2.3)$$

where:

C_p is the pad capacitance

h is the thickness of the substrate

e_{eff} is the effective dielectric constant of the substrate

l is the length of the pad

W_{eff} is the effective corrected transmission line width

i. **Resonant frequency:** The resonant frequency of an antenna is the frequency at which electromagnetic waves can be transmitted or received by the antenna. The resonant frequency of the quasi lumped element resonator antenna is calculated using the following expression (Ain and Hassan, 2004)

$$f = \frac{1}{2*\pi*\sqrt{L*\frac{C_{P1}C_{P2}}{C_{P1}+C_{P2}}+C}} \quad (2.4)$$

where:

f is the resonant frequency of the antenna
L is the inductance of the inductor
C_{p1}, is the pad capacitance

C is the capacitance of the interdigital capacitor
The parameters e_{eff} and w_{eff} are expressed mathematically as:

$$e_{eff} = \frac{e_r+1}{2} + \frac{e_r-1}{2} * \left(1 + 10 * \frac{\square}{w}\right)^{-\frac{1}{2}} \quad (2.5)$$

$$w_{eff} = w + \left[\frac{4*e}{\sqrt{(\frac{t}{\square})^2 + (\frac{1}{\pi(\frac{w}{t}+1.10)})^2}} \right] \qquad (2.6)$$

where:

w is the width of the pad

h is the height of the substrate

t is the thickness of the conductor

e is 2.713

w_{eff} is the effective corrected transmission line width

e_r is the dielectric constant of the substrate

Given all this parameters, the Quasi lumped element resonator antenna can be designed

Conclusion

This paper presented Lumped Elements and its Existence in Quasi Lumped Element Resonator Antenna. Lumped Elements was described and the formulas for describing those component Lumped Element parts were extracted from literatures. The Quasi Lumped Element Resonator Antenna is designed based on some lumped element component parts. Its resonant frequency is not proportional to the length of the antenna but the constitution of the lumped elements.

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